Simple Implementable Financial Policy Rules*

Mauricio Calani C.[1] Javier Moreno[1] Marco Piña[2]

[1] Central Bank of Chile
[2] University of Virginia

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Abstract

How important, for welfare, is the counter-cyclical capital buffer (CCyB) relative to other —higher and more permanent— bank capital requirements? While there is better understanding of the effect of a-cyclical higher capital requirements on banks' resilience and credit supply, much less is known about the marginal effects of introducing a macroprudential counter-cyclical capital requirement. In this paper, we study and rank the welfare gains of introducing several simple and implementable financial policy (CCyB) rules that co-exist with monetary policy. We find that the institutional design of the financial-policy instruments matters. In particular, a zero lower bound on the CCyB interacts with its counter-cyclical nature and provides a rationale for a positive *neutral* level. We build our analysis based on a quantitative macro-banking model with two main frictions, nominal rigidities and financial frictions, which we estimate for Chile.

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*Calani: mcalani@bcentral.cl, Moreno: jmoreno@bcentral.cl, Piña: wju9nq@virginia.edu. We thank comments from Rodrigo Alfaro, Paula Beltrán (discussant) and Saki Bigio.

1 Introduction

The 2008 financial crisis put forward the importance of financial intermediation, mainly through banking, in the potential origination and amplification of shocks to the macroeconomy. This observation catalyzed both, research on macro-financial linkages, and re-assessment of banking regulation. The latter materialized in the package of reforms we know as Basel III; with one of its main objectives being the incorporation of a system-wide approach to financial risk assessments, and financial policy; thereby explicitly introducing a macroprudential perspective to banks' capital regulation. Basel III introduces two buffers in this direction; the capital conservation buffer (CCoB) and the countercyclical capital buffer (CCyB) (Financial Stability Institute, 2019)[1]. While the CCoB has more automatic guidelines for its replenishment in case of loss-related draw downs, the CCyB can be activated and deactivated according to the decision of the authority. That is, the CCyB is a macroprudential tool. In this paper we examine the implications of different rules guiding this decision in terms of welfare and banks' resilience, how they interact with monetary policy, and emphasize the implications of the institutional design on the adequacy of a positive neutral level of CCyB.

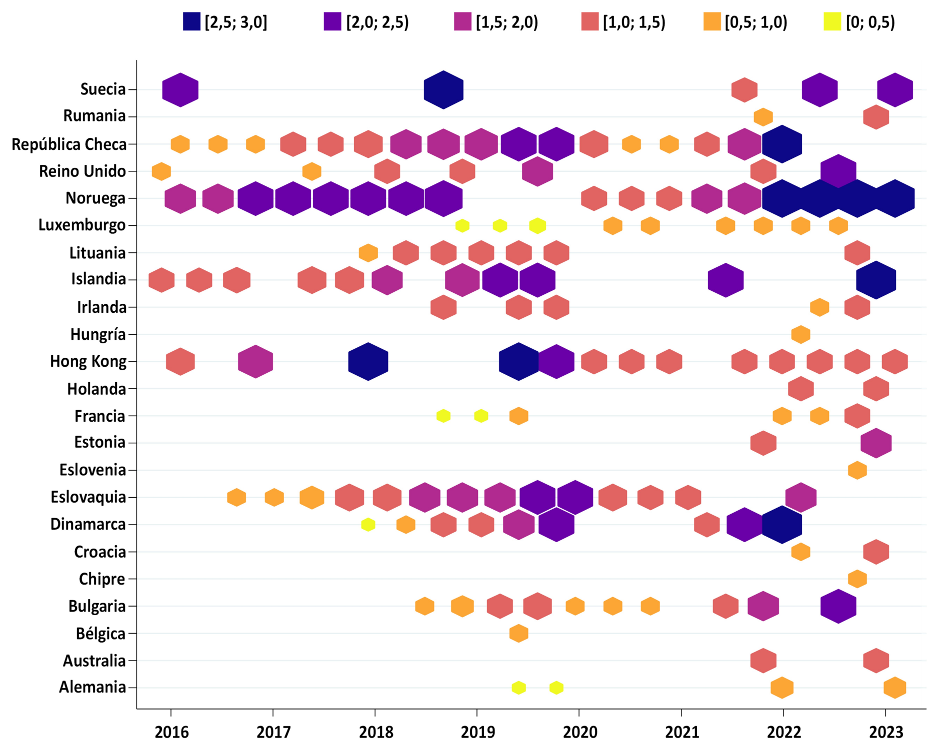
In order to comprehensively analyze the macroeconomic implications of different CCyB designs, we build a macro-banking model with two main inefficiencies as in Carrillo et al. (2021). Monetary policy addresses inefficiencies from staggered pricing by monopolistic input producers, and Financial policy addresses inefficiencies from financial frictions in the form of costly state verification. Drawing on the results of Carrillo et al. (2021) we abstract from a one-tool for two-objectives policy, and instead start from the Tinbergen rule. Our model includes both a monetary policy rule, and a countercyclical capital requirement rule, and features three levels of default by different agents in the economy, including the banking sector, as in Clerc et al. (2014). Hence our model is rich enough to analyze the interaction of monetary and financial policy, yet parsimonious enough to calculate welfare of different policy regimes. In particular, our model is based on a simplified version Calani et al. (2022), one of the main models used at the Central Bank of Chile. Notably, in the financial side, this model features financial frictions as in Bernanke et al. (1999) and Clerc et al. (2014); long term debt as in Woodford (2001a); and a bank-related friction in which depositors do not price bank default risk at the margin, as in Mendicino et al. (2018) and Mendicino et al. (2020). Our model is more appropriate for small open economies with both monetary and financial policies, in which bank credit can be short- and long-term.

The literature on the effects of banks' capital requirements on financial and real variables, has grown significantly in the past years, in tandem with the number of countries adopting and implementing capital regulation, and the availability of micro-data. However, at least on its aggregate consequences, most of the focus of the literature has emphasized the effects of the higher levels of capital requirements. The main trade-off of higher, a-cyclical, capital requirements weights lower systemic risk —measured as banking sector default probability— and lower activity in credit and the ensuing lower economic activity (Van den Heuvel, 2008; Clerc et al., 2014; Mendicino et al., 2018, 2020). Our paper shares this main feature, but instead, its focus is on cyclical considerations of capital regulation,

[1]Both capital buffers must be met with Common Equity Tier 1 (CET1) capital only. The CCoB is meant to give banks and additional layer of usable capital when idiosyncratic losses are incurred. The CCyB is meant to be raised when system-wide risks, usually associated with high credit growth is perceived to become more important. Both buffers range from 0% to 2.5%.

i.e. the design of a CCyB rule and its macroeconomic effects. Thus, our paper is more related to Carrillo et al. (2021) and Malherbe (2020). We explore different implementable, simple, policy rules in terms of their welfare implications, exploring the relationship with monetary policy. Notably, we find that simply following a credit-gap rule may not be optimal.

Figure 1: Countercyclical capital buffer activation across countries



Note.– This figure reports activation of countercyclical buffer (CCyB) by date and size of requirement. Each hexagon shows the current level of CCyB. No hexagon means deactivated CCyB. Source: Financial Stability Report CBC 2023-S1

Further, the experience from the Covid-19 pandemic suggests that there might be important differences between CCoB and CCyB usability. In particular, banks might be reluctant to exhaust CCoB (Basel Committee on Banking Supervision, 2022), and instead might want to comply with capital requirements deleveraging. In contrast, a system-wide deactivation of the countercyclical capital buffer by instruction of the supervisor, would not attract adverse market reaction or stigma on any particular bank, and might better accomplish its countercyclical objective. Notably, before the Covid-19 pandemic many juristictions had activated the CCyB, and deactivated it in early 2020 (see Figure 1). By the end of 2021, mostly the same economies started activating this buffer again, suggesting that its deactivation was useful during the worst moment of the sanitary crisis.

By design, however, the CCyB ranges from 0 to 2.5 percent of risk weighted assets (RWA), which implies that if a shock which would be better addressed by deactivating the CCyB, hits the economy, and this instrument is currently not activated, then much of its benefits are not grasped. This mechanism provides a rationale for setting a positive neutral level in case deactivation is suddenly required. We explore this issue quantitatively.

Using a quantitative model estimated with Chilean data, we explore several simple and implementable financial policy rules in terms of welfare differences (summarized in consumption equivalent terms). We find that, consumption equivalence is decreasing in the level of neutral CCyB, the lower and upper limits of 0 and 250bp are binding, and that rules put weight on future expected realizations of endogenous variables perform no better than rules that respond quickly to shocks. This goes in sharp contrast to conventional wisdom and efforts to forecast the financial cycle as a useful indicator for setting the CCyB.

The document is structured as follows. In Section 2 we present a detailed description of the theoretical structure of the model. Section 3.2 describes the estimation of the model, the calibration, the choice of priors and presents the results. Section 4 presents the results. Section 5 concludes.

1.1 Related Literature

Distinguishing features. [TO BE COMPLETED]

This is a DSGE model following closely ... informing the topics in ...

Long-term mortgage debt and the role of face-value terms for the decision of default of impatient households.

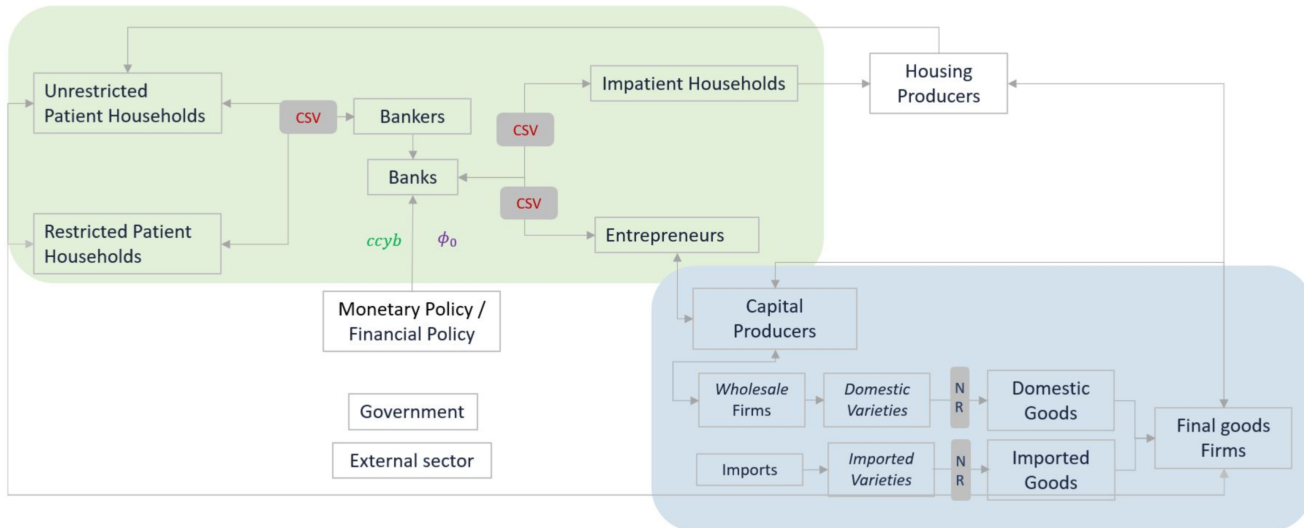
2 A Small Open Economy Model with Nominal and Financial Frictions

Our aim is to study the implications of (simple and implementable) Financial Policy Rules. Our analysis is based on a rich DSGE model featuring two main inefficiencies: nominal rigidities and financial frictions in the form of costly-state verification (CSV). We introduce CSV as in Bernanke et al. (1999) in three layers of the model, following Clerc et al. (2014) to introduce explicitly the notion of default, notably banking-system default probability. We depart from Clerc et al. (2014) by enriching our model to incorporate sticky prices and a role for monetary policy, as much of our analysis builds on welfare implications from taming business-cycle volatility, to which the role of monetary policy is first-order relevant. Thus we can compare different specifications of Financial Policy Rules at the margin, considering its interactions with monetary policy.

Figure 2 shows a sketch of agents adnn interactions in the model. Households are divided into two groups: patients and impatients, who in equilibrium, save and borrow respectively. Patient households can be "unrestricted", and have access to save in short or long-term assets, or "restricted", and be able to save only in short-term instruments[2]. Impatient households borrow resources from banks to finance housing purchases, subject to CSV and can thus default. Households negotiate their wages through unions. Entrepreneurs are the sole owners of productive capital, who finance their capital investment through banking loans, also subject to CSV. Bankers are the owners of bank equity, which in turn finance entrepreneurs and impatient households. From the production side, we introduce capital producers, housing-good producers, and productive firms related to the production of the final good. Wholesale firms produce domestic good varieties, which are combined with imported good varieties produced by importers. Final good producers combine domestic and imported goods. There is a monetary and financial authority besides a government with balanced fiscal budget.

[2]This distinction follows from Andres et al. (2004) and Chen et al. (2012) to introduce market segmentation and preferred habitat as in Vayanos and Vila (2009)

Figure 2: Graphical illustration of agents and frictions of the model



Note.– CSV stands for costly state verification and NR stands for nominal rigidities. Green box emphasizes the financial modules of the model, which are directly affected by Financial Policy. The blue box emphasizes the more standard New-Keynesian modules of the model, more directly affected by nominal rigidities and for which monetary policy is directly relevant.

There are two main sources of inefficiencies in this economy, nominal rigidities and financial frictions. Monetary Policy and Financial Policy are motivated by these two. The aim of this paper is to characterize the aggregate and welfare effects of different Financial Policy Rules. Next, we outline the main components of the model emphasizing those important to our results or distinctive in this model, leaving more standard components to be explained in detail in Appendix A

2.1 Households

Preferences depend on consumption of a final good (C_t), housing services from housing stocks (H_{t-1})—both relative to external habits—, and leisure. Households can differ in terms of their discount factor, being patient or impatient. Patient households can further be grouped into Restricted-Patient, and have access only to long-term assets, and Unrestricted-Patient who can access both short- and long-term assets. However, they can save in the long-term asset at a cost which is proportional to the ratio of their holdings of long-term instruments.

In equilibrium (restricted and unrestricted) patient households save. Short term assets include one-period deposits in banks, one-period government bonds, and one-period foreign bonds denominated in US dollars. We model long-term debt as instruments that pay geometric-decaying coupons as in Woodford (2001a). Long-term bonds can be issued either by the sovereign or by banks.

Also, in equilibrium, impatient households borrow from banks to finance their purchases of housing goods, subject to a financial friction: costly state verification (CSV). As the project to be financed is the purchase of housing good, it serves as collateral and its price is subject to an idiosyncratic shock which can trigger default. In states when the amount of contracted debt is higher than the value of the house, households default. Indeed, one of the reasons that we choose to model financial frictions through CSV as in Bernanke et al. (1999), is that default is an object that exists in equilibrium, and can vary in time Clerc et al. (2014). These mortgage loans are long-term obligations subject to a small transaction cost in case households need to adjust their debt levels (renegotiation).

2.2 Entrepreneurs

Entrepreneurs are the sole owners of productive capital K_t, which they rent to firms for the production of intermediate goods. They live two periods. In the second period they draw utility from transferring part of their wealth to households as dividends and leaving bequest to the next generation of entrepreneurs (initial net worth). This implies that entrepreneurs will not save their way out of requiring external financing from banks[3].

In the first period, entrepreneurs receive the bequests from the previous generation N_t^e, and maximize expected second period wealth, Ψ_{t+1}^e, by choosing purchases of capital at nominal price Q_t^K, and simultaneously the amount of commercial borrowing L_t^F from commercial banks (F-banks, henceforth).

$$Q_t^K K_t = N_t^e + L_t^F \tag{1}$$

Borrowing is also subject to CSV. After deciding the level of investment K_t in period one, entrepreneurs receive an idiosyncratic shock ω_{t+1}^e to the efficiency units of capital in period two, which affects their ability to pay their debt to banks[4]. This shock is only observable to entrepreneurs. Banks can verify if the reported ω_{t+1}^e is true at a cost μ. If the entrepreneur honors her debt she pays pre-set amount $R_t^F L_t^F$. If she defaults, the bank pays the verification cost and seizes all capital. This lending contract is a standard-debt-contract. It induces truth telling from the entrepreneur and minimizes the verification cost.

Then, second period entrepreneur's wealth is the proceeds from renting capital R_{t+1}^k and selling depreciated capital at price Q_{t+1}^K, minus debt repayment, only if this difference is positive.

$$\Psi_{t+1}^e = \max\left[\omega_{t+1}^e \left(R_{t+1}^k + (1-\delta_K)\, Q_{t+1}^K\right) K_t - R_t^L L_t^F,\ 0\right] \tag{2}$$

Limited liability defines a threshold $\overline{\omega}_{t+1}^e$ for ω_{t+1}^e, below which the entrepreneur defaults. This conveniently defines a default probability $PD_t^e = F_e(\bar{\omega}_t^e)$ for commercial loans.

In equilibrium the profitability of the project is split between lender and borrower. The share of the gross return that goes to the entrepreneur is $[1-\Gamma_e(\bar{\omega}_{t+1}^e)]$, and the share of gross return that goes to the bank is $\Gamma_e(\bar{\omega}_{t+1}^e)$. Banks subtract from this share, the verification costs stemming from non-performing loans, $\left(\bar{\omega}_{t+1}^e\right)$. Then, their net share of return is $\Gamma_e\left(\bar{\omega}_{t+1}^e\right) - \mu_e G_e\left(\bar{\omega}_{t+1}^e\right)$. We can re-write equation (2) using this notation and the accounting

[3]This part of the model follows closely Clerc et al. (2014) who also model a two-period living entrepreneur subject to CSV as in Bernanke et al. (1999)

[4]The ω_{t+1}^e is assumed to be log-normal as in Bernanke et al. (1999). More details in Appendix A

identity (1), and write the problem of the entrepreneur in t as

$$\max_{\bar{\omega}^e_{t+1},K_t} \mathbb{E}_t \left\{\Psi^e_{t+1}\right\} = \mathbb{E}_t \left\{\left[1-\Gamma_e\left(\bar{\omega}^e_{t+1}\right)\right] R^e_{t+1} Q^K_t K_t\right\}, \quad s.t.$$

$$\mathbb{E}_t \left\{\left[1-\Gamma_F\left(\bar{\omega}^F_{t+1}\right)\right]\left[\Gamma_e\left(\bar{\omega}^e_{t+1}\right)-\mu_e G_e\left(\bar{\omega}^e_{t+1}\right)\right] R^e_{t+1} Q^K_t K_t\right\} \geq \bar{\rho}_t \phi_F L^F_t, \tag{3}$$

where equation (3) is the participation constraint for banks. The first term in brackets in the LHS of (3) will become clearer later, but it comes the fact that another participation constraint applies also to the owner of bank equity–the banker. The rest of the LHS is the net return for lending to entrepreneurs. The RHS of the same equation is the demanded return $\bar{\rho}_t$ for commercial bank equity $E^F_t = \phi_F L^F_t$, where ϕ_F is the capital requirement for commercial banks.

2.3 Bankers and Banks

Bankers. Just as entrepreneurs, they live two periods and have exclusive access to the opportunity of investing their wealth as banks' inside equity capital. In the first period, the banker receives a bequest N^b_t from the previous generation and must distribute it between two types of banks: banks specializing in corporate loans (F banks) and in housing loans (H banks). Denote inside equity in each E^F_t and E^H_t, respectively. This allocation, together with realized return ρ^j_{t+1} on each j bank, determines second period total wealth,

$$\Psi^b_{t+1} = \rho^F_{t+1} E^F_t + \xi^{b,roe}_t \rho^H_{t+1}\left(N^b_t - E^F_t\right)$$

where $\xi^{b,roe}_t$ is a relative profit shock. As the banker chooses equity allocation in the first period, her problem is to maximize $\mathbb{E}_{\mathrm{t}}\left\{\Psi^b_{t+1}\right\}$ which results in the following to hold:

$$\mathbb{E}_{\mathrm{t}}\left\{\rho^F_{t+1}\right\} = \mathbb{E}_{\mathrm{t}}\left\{\xi^{b,roe}_t \rho^H_{t+1}\right\} = \bar{\rho}_t$$

where $\bar{\rho}_t$ denotes banks' required expected gross rate of return on equity investment undertaken at time t.

In the second period the banker decide how to distribute his wealth Ψ^b_{t+1} between dividends to households and bequests N^b_{t+1} to the next generation.

Banks. Banks are projects that invest in credit portfolios, financed with internal equity of bankers and households deposits or holdings of (long-term) bank-bonds. In particular, the balance sheet of bank-F is given by

$$L^F_t = E^F_t + D^F_t$$

and balance sheet of banks of class H is given by

$$Q^L_t L^H_t = E^H_t + Q^{BB}_t BB_t$$

Capital requirements are given by $E_t^F \geq \phi_F L_t^F$, $E_t^H \geq \phi_H Q_t^L L_t^H$, which are binding in equilibrium. We assume a continuum of banks of class $j = \{F, H\}$, with ex-post profits Π_{t+1}^j defined by:

$$\Pi_{t+1}^F = \max\left[\omega_{t+1}^F \tilde{R}_{t+1}^F L_t^F - R_t^D D_t^F, 0\right], \qquad \Pi_{t+1}^H = \max\left[\omega_{t+1}^H \tilde{R}_{t+1}^H Q_t^L L_t^H - R_{t+1}^{BB} Q_{t+1}^{BB} BB_t, 0\right]$$

where $\tilde{R}_{t+1}^j$ is the realized return on a well-diversified portfolio of loans to entrepreneurs or households, R_t^D is the interest rate on deposits, and Q_t^L and Q_t^{BB} are the price of long-term mortgage loans and bank bonds, respectively. Also, let ω_{t+1}^j denote an idiosyncratic portfolio return shock, which is i.i.d across banks of class j with a cdf of $F_j(\omega_{t+1}^j)$ and pdf $f_j(\omega_{t+1}^j)$. Limited liability for bankers defines thresholds $\bar{\omega}_{t+1}^j$:

$$\bar{\omega}_{t+1}^F \equiv \frac{R_t^D D_t^F}{\tilde{R}_{t+1}^F L_t^F}, \qquad \bar{\omega}_{t+1}^H \equiv \frac{R_{t+1}^{BB} Q_{t+1}^{BB} BB_t}{\tilde{R}_{t+1}^H Q_t^L L_t^H}$$

Similar to households and entrepreneurs, let $\Gamma_j(\bar{\omega}_{t+1}^j)$ denote the share of gross returns that goes to the creditor; in this case, depositors or bond holders, implying that $[1 - \Gamma_j(\bar{\omega}_{t+1}^j)]$ is the share that the bankers will keep as profits. We also define $G_j(\bar{\omega}_{t+1}^j)$ as the share of defaulting j banks, and thus $\mu_j G_j(\bar{\omega}_{t+1}^j)$ is the total verification cost of bank j default.

Finally, we are in position to define the realized rate of return of equity invested in a bank of class j:

$$\rho_{t+1}^j = \left[1 - \Gamma_j\left(\bar{\omega}_{t+1}^j\right)\right] \frac{\tilde{R}_{t+1}^j}{\phi_j} \tag{4}$$

2.4 Capital and Housing goods producers

As in Clerc et al. (2014), we model perfectly competitive capital-producing and housing-producing firms. Both, owned by households. They produce new units of capital and housing from the final good and sell them to entrepreneurs and households respectively. We depart from Clerc et al. (2014) by assuming time-to-build frictions in housing investment.

Capital goods. There is a continuum of competitive capital-producer firms who buy an amount I_t of final goods at price P_t and use their technology to satisfy the demand for new capital goods not covered by depreciated capital. New units of capital are sold at price Q_t^K. As is usual in the literature, we consider quadratic investment adjustment costs in the accumulation of capital:

$$K_t = (1 - \delta_K) K_{t-1} + \left[1 - \frac{\gamma_K}{2}\left(\frac{I_t}{I_{t-1}} - a\right)^2\right] \xi_t^i I_t$$

where γ_K controls the adjustment cost, and ξ_t^i is a shock to investment efficiency.

Housing goods. Housing good producers are subject to investment adjustment costs and time-to-build as in Kydland and Prescott (1982) and Uribe and Yue (2006). A continuum of competitive housing firm producers choose housing investment I_t^{AH} in period t, which will increase housing stock N_H periods later: the time it takes to build.[5] Thus, the law of motion for the aggregate stock of housing in H_t will consider projects authorized N_H periods before in interaction with adjustment costs,

$$H_t = (1-\delta_H) H_{t-1} + \left[1 - \frac{\gamma_H}{2}\left(\frac{I^{AH}_{t-N_H}}{I^{AH}_{t-N_H-1}} - a\right)^2\right] \xi^{ih}_{t-N_H} I^{AH}_{t-N_H} \tag{5}$$

where ξ^{ih}_t is a shock to housing investment efficiency. Time-to-build implies that firm's effective expenditure is spread out during the periods that new housing is being built. In particular, the amount of final goods purchased (at price P_t) by the firm in t to produce housing is given by

$$I^H_t = \sum_{j=0}^{N_H} \varphi^H_j I^{AH}_{t-j}$$

Where φ^H_j (the fraction of projects authorized in period $t-j$ that is outlaid in period t) satisfies $\sum_{j=0}^{N_H} \varphi^H_j = 1$ and $\varphi^H_j = \rho^{\varphi H} \varphi^H_{j-1}$.[6] The representative housing producer chooses how much to authorize in new projects I^{AH}_t in order to maximize the discounted utility of its profits.

2.5 Final good producing firms

The supply side of the economy is composed by different types of firms, all owned by the households. Monopolistically competitive unions act as wage setters, selling household's differentiated varieties of labor supply n_{it} to a perfectly competitive firm, which packs these varieties into a composite labor service $\widetilde{n}_t$. There is a continuum of monopolistically competitive firms producing different varieties j of a home good Y^H_{jt}, using wholesale good X^Z_t as input; a set of monopolistically competitive firms that import a homogeneous foreign good M_t to transform it into varieties, Y^F_{jt}; and three groups of perfectly competitive firms that aggregate products: one packing different varieties of the home good into a composite home good, Y^H_t, one packing the imported varieties into a composite foreign good, Y^F_t, and, finally, another one that bundles the composite home and foreign goods to create a final good, Y^C_t. This final good is purchased by households (C^P_t,C^I_t), capital and housing producers (I^K_t,I^H_t), and the government (G_t).

Final goods. A representative final-goods firm demands composite home good X^H_t, and composite foreign goods X^F_t, and combines them according to the following technology:

$$Y^C_t = \left[\omega^{1/\eta}\left(X^H_t\right)^{1-1/\eta} + (1-\omega)^{1/\eta}\left(X^F_t\right)^{1-1/\eta}\right]^{\frac{\eta}{\eta-1}} \tag{6}$$

where $\omega \in (0,1)$ controls home bias and $\eta > 0$ measures the substitutability between domestic and foreign goods. The price of the final good is P_t, and P^H_t and P^F_t denote the prices of the home composite and foreign composite goods, respectively.

[5] Notice that if $N_H = 0$, the structure is symmetric to the capital producers.

[6] Notice that $\rho^{\varphi H} > 1$ implies that expenditure for any authorized project is back-loaded (increasing over time), while the converse is true for $\rho^{\varphi H} < 1$.

Home composite goods. A representative home composite goods firm demands all $j \in [0,1]$ varieties of intermediate home goods in amounts X_{jt}^H, and combines them according to the technology

$$Y_t^H = \left[\int_0^1 \left(X_{jt}^H\right)^{\frac{\epsilon_H - 1}{\epsilon_H}} dj\right]^{\frac{\epsilon_H}{\epsilon_H - 1}} \tag{7}$$

with $\epsilon_H > 0$. Let P_{jt}^H denote the price of the home good of variety j. The firm maximizes its profits $\Pi_t^H = P_t^H Y_t^H - \int_0^1 P_{jt}^H X_{jt}^H dj$ choosing input demands X_{jt}^H, subject to (7) and taking prices as given.

Intermediate Home Goods of Variety j. There is measure one of firms, that demand domestic wholesale goods X_t^Z and differentiate into j intermediate home good varieties Y_{jt}^H. To produce one unit of variety j, firms need one unit of input according to

$$\int_0^1 Y_{jt}^H dj = X_t^Z \tag{8}$$

The firm producing variety j satisfies the demand from the home-composite producing firm Y_t^H, and has monopoly power for its variety. Given (8), the nominal marginal cost in terms of the composite good price is given by $P_t^H mc_{jt}^H$. As every firm buys their input from the same wholesale market, all of them face the same nominal marginal costs

$$P_t^H mc_{jt}^H = P_t^H mc_t^H = P_t^Z \tag{9}$$

Firm j chooses its price P_{jt}^H to maximize profits, taking marginal costs in as given. In setting prices, the firm are subject to Calvo-type nominal rigidities, whereby each period the firm can change its price optimally with probability $1 - \theta_H$, and if it cannot optimally change its price, it indexes its previous price according to a weighted product of past and steady state inflation with weights $\kappa_H \in [0,1]$ and $1 - \kappa_H$ respectively.

Wholesale Domestic Goods. A representative firm produces a homogeneous wholesale home good, combining capital K_{t-1} and composite labor $\tilde{n}_t$ according to the following technology

$$Y_t^Z = z_t K_{t-1}^{\alpha} \left(A_t \tilde{n}_t\right)^{1-\alpha} \tag{10}$$

with capital share $\alpha \in (0,1)$, an exogenous stationary technology shock z_t and a non-stationary technology A_t shock. Notably, the firm faces adjustment costs of labor. Profit maximization implies that the price of this wholesale good is equated to marginal cost.

Foreign composite goods. Like with home composite goods, a representative firm demands foreign goods of all $j \in [0,1]$ varieties in amounts X_{jt}^F and combines them into Y_t^F according to the following technology with $\epsilon_F > 0$.

$$Y_t^F = \left[\int_0^1 \left(X_{jt}^F\right)^{\frac{\epsilon_F - 1}{\epsilon_F}} dj\right]^{\frac{\epsilon_F}{\epsilon_F - 1}} \tag{11}$$

Intermediate foreign goods of variety j. Importing firms buy an amount M_t of a homogeneous foreign good at the price $P_t^{M\star}$ abroad, and convert this good into varieties Y_{jt}^F that are sold domestically. Total imports are $\int_0^1 Y_{jt}^F dj$. We assume that the import price level $P_t^{M\star}$ co-integrates with the foreign producer price level $P_t^\star$, i.e., $P_t^{M\star} = P_t^\star \xi_t^m$, where ξ_t^m is a stationary exogenous process. As it takes one unit of the foreign good to produce one unit of variety j, nominal marginal costs in terms of composite goods prices are common across varieties

$$P_t^F mc_{jt}^F = P_t^F mc_t^F = S_t P_t^{M\star} = S_t P_t^\star \xi_t^m \tag{12}$$

Producer of variety j has monopoly power for its variety. Given marginal costs, the firm producing variety j chooses its price P_{jt}^F to maximize profits. In setting prices, the firm faces a Calvo-type problem similar to domestic firms. The firm can change its price optimally with probability $1-\theta_F$, else it indexes its previous price according to a weighted product of past and steady state inflation.

The model then features inefficiencies due to staggered pricing by monopolistic input producers in two markets; the home and foreign intermediate goods markets. This nominal frictions motivate the existence of monetary policy, as in the benchmark NK model.

Wages. Recall that demand for productive labor is satisfied by perfectly competitive packing firms that *demands* all varieties $i \in [0,1]$ of labor services in amounts $n_t(i)$ and combine them in order to produce composite labor services $\widetilde{n}_t$

$$\widetilde{n}_t = \left[\int_0^1 n_t(i)^{\frac{\epsilon_W - 1}{\epsilon_W}} di\right]^{\frac{\epsilon_W}{\epsilon_W - 1}}, \qquad \epsilon_W > 0. \tag{13}$$

Differentiated labor $n_t(i)$ is supplied by a continuum of monopolistically competitive unions who set wages subject to the demand of labor-packing firms, and to nominal rigidities à la Calvo. These unions allocate labor demand uniformly across patient and impatient households, so $n_t^P(i) = n_t^I(i)$ and $n_t^P(i) + n_t^I(i) = n_t(i)\ \forall i, t$, with $n_t^P(i) = \wp_U n_t^U(i) + (1-\wp_U) n_t^R(i)$, which also holds for the aggregate n_t^P, n_t^I and n_t.

Commodities. We assume the country receives an exogenous and stochastic endowment of commodities Y_t^{Co}. Moreover, these commodities are not consumed domestically but entirely exported. Therefore, the entire production is sold at a given international price $P_t^{Co\star}$, which is assumed to evolve exogenously. We further assume that the government receives a share $\chi \in [0,1]$ of this income and the remaining share goes to foreign agents.

2.6 Fiscal and Monetary policies

Fiscal Policy. The government consumes an exogenous stream of final goods G_t, pays (through an insurance agency IA_t) for deposits and bonds defaulted by banks, levies lump-sum taxes on patient households T_t^P, issues one-period bonds BS_t^G and long-term bonds BL_t^G, and receives revenue from commodity exports $\chi S_t P_t^{Co\star} Y_t^{Co}$. The government satisfies the following period-by-period constraint where sources of funds (LHS) equate uses of funds (RHS):

$$T_t - BS_t^G - Q_t^{BL} BL_t^G + \chi S_t P_t^{Co\star} Y_t^{Co} = P_t G_t - R_{t-1} BS_{t-1}^G - R_t^{BL} Q_t^{BL} BL_{t-1}^G + IA_t \tag{14}$$

As in Chen et al. (2012), we assume that the government control the supply of long-term bonds according to a simple rule given by an exogenous AR(1) process on BL_t^G.

Monetary Policy. In turn, following Garcia et al. (2019) monetary policy is follows a Taylor Rule of the form

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\alpha_R} \left[\left(\frac{(1-\alpha_E)\,\pi_t + \alpha_E \mathbb{E}_t\left\{\pi_{t+4}\right\}}{\pi_t^T}\right)^{\alpha_\pi} \left(\frac{GDP_t/GDP_{t-1}}{a}\right)^{\alpha_y}\right]^{1-\alpha_R} e_t^m \tag{15}$$

where $\alpha_R \in [0,1)$, $\alpha_\pi > 1$, $\alpha_y \geq 0$, $\alpha_E \in [0,1]$ and where π_t^T is an exogenous inflation target and e_t^m an i.i.d. shock that captures deviations from the rule.[7]

2.7 Financial Policy

This paper's main contribution to the literature is the examination of how financial policy impacts allocations, prices and ultimately welfare. Financial policy takes the form of counter-cyclical capital (CCyB) requirements. We explore different specifications for such a rule in this paper, among those that are simple and implementable. The CCyB rule depends on its own lag and some endogenous variable X_t as well as its expected value at some future *horizon*. We develop more on the exact functional forms, and explore the parameters governing this policy rule in section 4.

$$\left(\frac{1+CCyB_t}{1+\overline{CCyB}}\right) = \left(\frac{1+CCyB_{t-1}}{1+\overline{CCyB}}\right)^{\theta_1} \left(\frac{(1-\alpha_E)X_t + \alpha_E \mathbb{E}(X_{t+horizon})}{X}\right)^{\theta_2} e_t^{req} \tag{16}$$

2.8 Rest of the world

Real exchange rate. Foreigners demand both, the home composite goods and the domestic commodity. The structure of the foreign economy is identical to the domestic economy, but the latter is assumed to be small relative to the foreign economy. This implies that the foreign producer price level $P_t^\star$ is identical to the foreign consumption-based price index. Further, let $P_t^{H\star}$ denote the price of home composite goods expressed in foreign currency. There are no transaction costs or other barriers to trade, so the law of one price holds separately for home composite goods and the commodity good, i.e. $P_t^H = S_t P_t^{H\star}$ and $P_t^{Co} = S_t P_t^{Co\star}$. Due to local currency pricing, a weak form of the law of one price holds for foreign composite goods, i.e., $P_t^F mc_t^F = S_t P_t^\star \xi_t^m$. The real exchange rate rer_t therefore satisfies

$$rer_t = \frac{S_t P_t^\star}{P_t} = \frac{P_t^F}{P_t}\frac{mc_t^F}{\xi_t^m} \tag{17}$$

Interest rate. The relevant foreign nominal interest rate is composed by an exogenous risk-free world interest rate R_t^W plus a country premium that decreases with the economy's net foreign asset position, expressed as a ratio of nominal GDP, as in

$$R_t^\star = R_t^W \exp\left\{-\frac{\phi^\star}{100}\left(\frac{S_t B_t^\star}{GDPN_t} - \bar{b}\right)\right\}\xi_t^R z_t^R \tag{18}$$

with $\phi^\star > 0$ and where ξ_t^R is an exogenous shock to the country premium.

[7] We do not need a time-varying target, so we will set it to a constant.

2.9 Market clearing and aggregation

This is a large model with many market clearing conditions: final gooods, intermediate goods, factor markets and financial asset markets.

Goods markets. In particular, markets must clear for goods,

$$Y_t^C = C_t^P + C_t^I + I_t + I_t^H + G_t + {}^{\Upsilon_t}/_{P_t} \tag{19}$$

where Υ_t includes final goods used in default costs: the resources lost by households recovering deposits at failed banks, the resources lost by the banks to recover the proceeds from defaulted bank loans by the recovery of deposits by the deposit insurance agency and adjustment costs.

In the market for the home and foreign composite goods we have, respectively,

$$Y_t^H = X_t^H + X_t^{H\star} \tag{20}$$

$$Y_t^F = X_t^F \tag{21}$$

while in the market for home and foreign varieties we have,

$$Y_{jt}^H = X_{jt}^H, \qquad \forall j$$

$$Y_{jt}^F = X_{jt}^F, \qquad \forall j$$

By the same token, in the market for the wholesale domestic good, we have $Y_t^Z = X_t^Z$. Finally, in the market for housing, demand from both households must equal supply from housing producers $H_t = H_t^P + H_t^I$

Factors of production. Labor and capital markets must also clear

Financial Assets. Deposits demand by banks and supply by patient households must equate

$$D_t^F = D_t^{Tot} \tag{22}$$

Similarly, the aggregate net holding of participating agents in bond markets are in zero net supply:

$$BL_t^{Pr} + BL_t^{CB} + BL_t^G = 0 \tag{23}$$

$$BS_t^{Pr} + BS_t^G = 0 \tag{24}$$

where BL_t^{CB} is an exogenous process denoting long-term bond purchases by the Central Bank.

Aggregate demand. GDP is defined as the sum of domestic absorption Y_t^C and trade balance, with nominal trade balance defined as

$$TB_t = P_t^H X_t^{H\star} + S_t P_t^{Co\star} Y_t^{Co} - S_t P_t^{M\star} M_t \tag{25}$$

Real GDP, in turn, is defined as

$$GDP_t = Y_t^{NoCo} + Y_t^{Co}$$

where non-mining GDP, Y_t^{NoCo}, is given by

$$Y_t^{NoCo} = C_t^P + C_t^I + I_t + I_t^H + G_t + X_t^{H\star} - M_t$$

and nominal GDP is defined as

$$GDPN_t = P_t \left(C_t^P + C_t^I + I_t + I_t^H + G_t \right) + TB_t \tag{26}$$

Note that by combining (26) with the zero profit condition in the final goods sector, i.e., $P_t Y_t^C = P_t^H X_t^H + P_t^F X_t^F$, and using the market clearing conditions for final and composite goods,(19), (21) and (20), GDP is seen to be equal to total value added (useful for the steady state):

$$\begin{aligned} GDPN_t &= P_t Y_t^C - \Upsilon_t + P_t^H X_t^{H\star} + S_t P_t^{Co\star} Y_t^{Co} - S_t P_t^{M\star} M_t \\ &= P_t^H X_t^H + P_t^F X_t^F - \Upsilon_t + P_t^H X_t^{H\star} + S_t P_t^{Co\star} Y_t^{Co} - S_t P_t^{M\star} M_t \\ &= P_t^H Y_t^H + S_t P_t^{Co\star} Y_t^{Co} + P_t^F X_t^F - S_t P_t^{M\star} M_t - \Upsilon_t \end{aligned}$$

Taking stock. The purpose of this brief sketch of the model is to inform the reader of the main structure and frictions/inefficiencies present in the model, not a full description of all equlibrium conditions and all details. The interested reader is referred to Appendix A which documents in detail the full model used in this paper, equilibrium conditions in stationary form, and the computation of the steady state.

3 Parameterization strategy and estimation results

The model parameters are calibrated and estimated. The calibrated parameters include those characterizing model dynamics for which we have a data counterpart, those drawn from related studies, and those chosen to match long-run ratios for Chile. In particular, we follow closely the calibration strategy from Garcia et al. (2019) and Clerc et al. (2014), as the models described there form the basis of this paper's framework. We estimate the non-calibrated parameters using Bayesian techniques as discussed below.

3.1 Calibration

Table 1 presents the values of the parameters related to the real sector of the economy that are either chosen from previous studies in the relevant literature or chosen in order to match exogenous steady state moments. The value of the parameters α, α_E, β_U, β_R, χ, ϵ_F, ϵ_H, ϵ_W, ω and π^T are taken from Garcia et al. (2019). We assume that the housing capital depreciation rate, δ_H is equal to the productive capital depreciation rate, δ_K, whose value is taken from Adolfson et al. (2013). The value for β_I is taken from Clerc et al. (2014).

Table 1: Calibration, Real Sector

Parameter	Description	Value	Source
α	Capital share in production function	0.34	Garcia et al. (2019)
α_E	Expected Inflation weight in Taylor Rule	0.50	Garcia et al. (2019)
α^{BSG}	Short-term govt. bonds as percentage of GDP	-0.40	Data: 2009-2019
α^{BLG}	Long-term govt. bonds as percentage of GDP	-4.50	Data: 2009-2019
β_U, β_R	Patient HH Utility Discount Factors	0.99997	Garcia et al. (2019)
β_I	Impatient Utility HH Discount Factor	0.98	Clerc et al. (2014)
δ_K	Capital Annual depreciation rate	0.01	Adolfson et al. (2013)
δ_H	Housing Annual Depreciation rate	0.01	Same as capital depreciation
ϵ_F	Elasticity of substitution among foreign varieties	11	Garcia et al. (2019)
ϵ_H	Elasticity of substitution among home varieties	11	Garcia et al. (2019)
ϵ_W	Elasticity of substitution among types of workers	11	Garcia et al. (2019)
ϵ_τ	Convergence speed towards SS Gov debt	0.10	Normalization
N_H	Time-to-build periods in housing goods	6	CBC 2018S2 Financial Stability Report
κ	Coupon discount in housing loans	0.975	10 years duration of loan contract
κ_{BL}	Coupon discount in long term government bonds	0.975	10 years bond duration
κ_{BB}	Coupon discount in long term banking bonds	0.95	5 years bond duration
π^T	Annual inflation target of 3%	$1.03^{1/4}$	Garcia et al. (2019)
$\rho_{\varphi h}$	Spending profile for long term housing investment	1	Even investment distribution
σ	Log Utility	1	Garcia et al. (2019)
υ	Strength of households wealth effect	0	No wealth effect
χ	Government share in commodity sector	0.33	Garcia et al. (2019)
ω	Home bias in domestic demand	0.79	Garcia et al. (2019)
$\wp_U$	Fraction of unrestricted patient households	0.70	Chen et al. (2012)
ω_{BL}	Ratio of long term assets to short assets	0.822	Chen et al. (2012)

The parameters that set the steady state value of short term and long term government bonds as a percentage of GDP, α^{BSG} and α^{BLG}, respectively, were calculated from data obtained from Depósito Central de Valores (DCV).[8] The parameters that determine the coupons' geometric decline of the long term housing debt, κ, and government bonds, κ_{BL}, are set so their duration is 10 years. The duration of the bank bonds, κ_{BB}, is set to 5 years.

The value used for the time that takes a house to be built, N_H is taken from the second semester of 2018 Financial Stability Report (FSR), equal to 6 quarters in order to match the average length of construction projects. We also assume an even investment spending profile for housing capital, consistent with a value of 1 for $\rho_{\varphi h}$. Following Garcia et al. (2019), we set the value of the parameter that determines the strength of the wealth effect, υ, to 0, to avoid undesired dynamics in the labor market.

For the calibration of the parameters related to the financial sector, shown in Table 2, the values of χ_b, χ_e, γ_{bh}, γ_d, μ_e, μ_F, μ_H and μ_I come from Clerc et al. (2014). The values for the parameters related to bank capital requirements, ϕ_F and ϕ_H, are set as the ratio between the average level of TIER I capital of over the risk weighted assets of the banking system from the year 2000 to the year 2020. In particular, we calculate 4.3% excess of TIER I capital in addition to legal 9.75%. For corporate banks we assume 100% weight in corporate loans, while for

[8] DCV is an entity that registers ownership of financial instruments take place in several exchange markets.

housing bank we assume 60% weight in housing loans.

Table 2: Calibration, Financial Sector

Parameter	Description	Value	Source
χ_b	Banks dividend policy	0.04	Clerc et al. (2015)
χ_e	Entrepreneurs dividend policy	0.05	Clerc et al. (2015)
γ_{bh}	Household cost bank bonds default	0.10	Clerc et al. (2015)
γ_d	Cost of recovering defaulted bank deposits	0.10	Clerc et al. (2015)
ϕ_F	Bank Capital Requirement (RWA)	0.14	Data (2000-2022)
ϕ_H	Bank Capital Requirement (RWA)	0.10	Data (2000-2022)

3.2 Estimation

We provide all model details in Appendix A, summarize equilibrium conditions in Appendix B and compute the non-stochastic steady state in Appendix C and D.The parameters whose values are not calibrated are estimated using Bayesian methods. The data for the estimation, described in Table 3, includes 25 macroeconomic and financial variables from between 2001Q3 and 2019Q3. Data for the real Chilean sector is obtained from the Central Bank of Chile's National Accounts database, while prices and labor statistics are obtained from the National Statistics Institute (INE). Finally, local financial data is obtained from the Financial Markets Committee (CMF), and foreign data is obtained from Bloomberg. Variables regarding the real sector are log-differentiated with respect to the previous quarter. All variables are demeaned. Our estimation strategy also includes i.i.d. measurement errors for all local observables with the exception of the policy rate. The variance of the measurement errors is calibrated to 10% of the variance of the corresponding observable, as is standard in the literature.

Table 3: Observable Data

This table summarizes the observable data time-series we feed the model for Bayesian estimation. The symbol $\Delta \log$ implies we take the log of the referred series, take first differences and subtract the mean. For all other variables we subtract the sample mean. Sources: INE, CBC, CMF, and Bloomberg

Non Financial		Financial	
$\Delta \log Y_t^{NoCo}$	Non mining real GDP	R_t^L	Comercial Loans interest Rate
$\Delta \log Y_t^{Co}$	Copper real GDP	R_t^I	Housing Loans Interest Rate
$\Delta \log C_t$	Total Consumption	R_t^D	Nominal Interest Rate on Deposits
$\Delta \log G_t$	Goverment Consumption	R_t^{LG}	10 Year BCP Rate
$\Delta \log I_t^K$	Real Capital Investment	$\Delta \log L_t$	Housing and Corporate Loan
$\Delta \log I_t^H$	Real Housing Investment	ROE_t	Banks ROE
$TB_t/GDPN_t$	Trade Balance-GDP Ratio	R_t^*	LIBOR
$\Delta \log N_t$	Total Employment	Ξ_t^R	EMBI Chile
$\Delta \log WN_t$	Nominal Cost of labor	rer_t	Real Exchange Rate
π_t	Core CPI	R_t	Nominal MPR
$\Delta \log y_t^*$	Real External GDP		
π_t^*	Foreign Price Index		
π_t^M	Imports Deflactor		
π_t^{Co*}	Nominal Copper Price		
π_t^H	Housing Price Index		

The posterior estimates are obtained using full information maximum likelihood estimation. To facilitate optimization, we scale shocks' standard deviations a similar order of magnitude in the posterior estimation (See Christiano et al. 2011). We choose the type of priors according to the related literature from distributions that have supported distributions consistent with the theoretical values expected for the parameters. In columns three, four and five of Table (4) we show the chosen prior distributions and prior distribution moments of the estimated values of the deep parameters. The sixth and seventh columns of the same table show the posterior mean and the 95% interval of the estimation. On Table 5 we show the estimation priors and results of the parameters related to shock variables. For all autocorrelation coefficient we use a beta distribution while for the standard deviation we use a inverse gamma distribution.

Table 4: Estimation

This table shows the first two moments of the prior distribution of estimated parameters, together with posterior mean and 95% credible intervals, based on maximum likelihood estimation and the Laplace approximation.

Parameter	Description	Prior			Posterior		
		Dist	Mean	St Dev	Mean	95% Inter	
α_π	Inflation weight in Taylor Rule	N	1.70	0.10	1.92	[1.76	2.08]
α_R	Lagged interest rate weight in Taylor Rule	β	0.85	0.03	0.77	[0.74	0.81]
α_W	Weight on past productivity on wage indexation	β	0.25	0.08	0.17	[0.04	0.29]
α_y	Output weight in Taylor Rule	N	0.13	0.08	0.13	[0.01	0.25]
η	Elasticity of subst. home and foreign goods	γ	1.00	0.25	0.97	[0.71	1.23]
$\eta_{\hat{C}}$	Elasticity of subst. consumption and housing goods	γ	1.00	0.25	0.12	[0.05	0.19]
η^*	Foreign demand elasticity of substitution	γ	0.25	0.08	0.19	[0.07	0.30]
γ_H	Housing investment adjustment cost parameter	γ	3.00	0.25	2.98	[2.48	3.49]
γ_K	Capital investment adjustment cost parameter	γ	3.00	0.25	2.95	[2.46	3.43]
γ_n	Labor adjustment cost parameter	γ	3.00	0.25	1.80	[1.46	2.13]
γ_L	Housing debt cost parameter	γ	0.1	0.09	0.29	[0.11	0.47]
κ_F	Weight on past inflation on foreign good indexation	β	0.50	0.08	0.67	[0.55	0.79]
κ_H	Weight on past inflation on home good indexation	β	0.50	0.08	0.76	[0.66	0.86]
κ_W	Weight on past inflation on wages indexation	β	0.85	0.03	0.85	[0.79	0.90]
ϕ^*	Country premium elasticity to NFA position	γ^{-1}	1.00	Inf	0.34	[0.16	0.52]
ϕ_c	Habit formation in good consumption	β	0.85	0.03	0.89	[0.86	0.92]
ϕ_{hh}	Habit formation in housing consumption	β	0.85	0.03	0.81	[0.75	0.86]
θ_F	Calvo param. foreign goods producers	β	0.50	0.08	0.72	[0.68	0.75]
θ_H	Calvo param. domestic goods producers	β	0.50	0.03	0.82	[0.80	0.84]
θ_W	Calvo param. wage setters	β	0.50	0.08	0.58	[0.51	0.65]
φ	Inverse Frisch elasticty	γ	7.50	1.50	8.37	[5.84	10.9]
μ_e	Monitoring cost of corporate loan default	β	0.30	0.05	0.45	[0.36	0.54]
μ_F	Monitoring cost of F bank default	β	0.30	0.05	0.37	[0.26	0.47]
μ_H	Monitoring cost of H bank default	β	0.30	0.05	0.30	[0.20	0.40]
μ_i	Monitoring cost of housing loan default	β	0.30	0.05	0.23	[0.14	0.32]
η_{ζ_L}	Term premium elasticity to relative bond liquidity	γ	0.15	0.03	0.14	[0.08	0.20]

4 Results

In this section we dig deeper in the features of the model. First we assess how the CCyB operates and its transmission mechanism. Next we quantify the implications of different Financial Policy (FP) rules for welfare. In particular, we consider rules that are simple and implementable from the policy-maker perspective. Using the results of this

Table 5: Estimation, exogenous variables AR1 processes

This table shows the first two moments of the prior distribution of estimated parameters, together with posterior mean and standard deviation, based on maximum likelihood estimation and the Laplace approximation. Note that some standard deviations are scaled by different factors to obtain posterior means that are in the same order of magnitude. All autocorrelations were estimated using the Beta distribution, while standard deviations using the inverse-gamma distribution.

Shock process	**A.R**	**Prior**		**Posterior**			**S.D.**	**Prior**		**Posterior**		
		Mean	**S.D**	**Mean**	**90% HPD**			**Mean**	**S.D**	**Mean**	**90% HPD**	
Non stat. productivity	ρ_a	0.25	0.08	0.37	[0.20	0.55]	$100 \times \sigma_a$	0.50	Inf	0.38	[0.26	0.51]
Monetary Policy	ρ_{e^m}	0.15	0.08	0.26	[0.06	0.46]	$1000 \times \sigma_{e^m}$	0.50	Inf	1.4	[1.03	1.77]
Government spending	ρ_g	0.75	0.08	0.75	[0.62	0.88]	$100 \times \sigma_g$	0.50	Inf	1.77	[1.46	2.09]
Copper price	$\rho_{p^{co}}$	0.75	0.08	0.89	[0.84	0.94]	$100 \times \sigma_{p^{co}}$	0.50	Inf	1.10	[0.90	1.30]
Foreign inflation	ρ_{π^*}	0.75	0.08	0.44	[0.37	0.52]	$100 \times \sigma_{\pi^*}$	0.50	Inf	2.20	[1.79	2.62]
Foreign interest rate	ρ_{R^W}	0.75	0.08	0.89	[0.84	0.94]	$1000 \times \sigma_{R^W}$	0.50	Inf	1.10	[0.84	1.36]
Entrepreneurs risk	ρ_{σ^e}	0.75	0.08	0.96	[0.93	0.99]	$100 \times \sigma_{\sigma^e}$	0.50	Inf	2.42	[1.77	3.07]
Corporate bank risk	ρ_{σ^F}	0.75	0.08	0.70	[0.56	0.85]	$10 \times \sigma_{\sigma^F}$	0.50	Inf	1.02	[0.46	1.59]
Housing bank risk	ρ_{σ^H}	0.75	0.08	0.77	[0.61	0.92]	$10 \times \sigma_{\sigma^H}$	0.50	Inf	0.23	[0.04	0.42]
Housing valuation risk	ρ_{σ^I}	0.75	0.08	0.92	[0.86	0.98]	$10 \times \sigma_{\sigma^I}$	0.50	Inf	5.39	[1.56	9.22]
Current consumption prefs.	ρ_ϱ	0.75	0.08	0.38	[0.28	0.49]	$10 \times \sigma_\varrho$	0.50	Inf	3.35	[1.78	4.91]
Housing consumption prefs	ρ_{ξ^h}	0.75	0.08	0.93	[0.90	0.95]	$10 \times \sigma_{\xi^h}$	0.50	Inf	1.42	[0.66	2.18]
Investment mg. eff.(K)	ρ_{ξ^i}	0.75	0.08	0.57	[0.42	0.72]	$10 \times \sigma_{\xi^I}$	0.50	Inf	0.69	[0.41	0.96]
Investment mg. eff.(H)	$\rho_{\xi^{ih}}$	0.75	0.08	0.88	[0.78	0.98]	$10 \times \sigma_{\xi^{ih}}$	0.50	Inf	1.75	[0.89	2.61]
Import prices	ρ_{ξ^m}	0.75	0.08	0.85	[0.76	0.93]	$100 \times \sigma_{\xi^m}$	0.50	Inf	2.56	[1.93	3.19]
Labor disutility	ρ_{ξ^n}	0.75	0.08	0.75	[0.60	0.89]	$10 \times \sigma_{\xi^n}$	0.50	Inf	3.86	[1.38	6.34]
Country premium	ρ_{ξ^R}	0.75	0.08	0.84	[0.75	0.92]	$1000 \times \sigma_{\xi^R}$	0.50	Inf	0.65	[0.50	0.79]
Banker dividend	$\rho_{\xi^{\chi_b}}$	0.75	0.08	0.82	[0.72	0.93]	$10 \times \sigma_{\xi^{\chi_b}}$	0.50	Inf	2.56	[1.93	3.19]
Entrepreneur dividend	$\rho_{\xi^{\chi_e}}$	0.75	0.08	0.45	[0.34	0.56]	$10 \times \sigma_{\xi^{\chi_e}}$	0.50	Inf	2.02	[1.53	2.51]
Banker required return	$\rho_{\xi^{roe}}$	0.75	0.08	0.83	[0.74	0.92]	$10 \times \sigma_{\xi^{roe}}$	0.50	Inf	0.37	[0.26	0.48]
Foreign demand	$\rho_{\xi^{y*}}$	0.85	0.08	0.90	[0.79	1.02]	$100 \times \sigma_{\xi^{y*}}$	0.50	Inf	0.24	[0.04	0.44]
Mining productivity	$\rho_{\xi^{yco}}$	0.85	0.08	0.80	[0.63	0.97]	$100 \times \sigma_{\xi^{yco}}$	0.50	Inf	3.23	[2.63	3.82]
Stat. productivity	ρ_z	0.85	0.08	0.84	[0.76	0.93]	$100 \times \sigma_z$	0.50	Inf	1.22	[0.91	1.53]
UIP shock	ρ_{ζ^u}	0.75	0.08	0.96	[0.93	0.98]	$1000 \times \sigma_{z_\tau}$	0.50	Inf	1.64	[0.76	2.52]
Liquidity costs	ρ_{ϵ^L}	0.75	0.05	0.76	[0.66	0.86]	$100 \times \sigma_{\epsilon^L}$	0.50	Inf	0.09	[0.02	0.17]

analysis we dig deeper in the implementability in consideration of lower and upper legal bounds considered in Basel III; 0 and 2.5%, respectively.

4.1 Transmission mechanism

For this exercise we explore the effect of activation of the CCyB in the estimated model using the simplest possible financial policy rule; one in which the buffer is activated only as an exogenous shock. Later we will consider more realistic and interesting rules. For now, let us consider the following rule

$$\left(\frac{1+CCyB_t}{1+\overline{CCyB}}\right) = \left(\frac{1+CCyB_{t-1}}{1+\overline{CCyB}}\right)^{\theta_1} e_t^{req} \tag{27}$$

where $0 < \theta_1 < 1$ is the persistence governing the CCyB, and is set equal to $\theta_1 = 0.917$ which is equivalent to a rule of mean-life of 8 quarters. Since an activation of the CCyB generates effects with financial origins, we will start with the financial transmission channels and then explain how they affect the real economy.

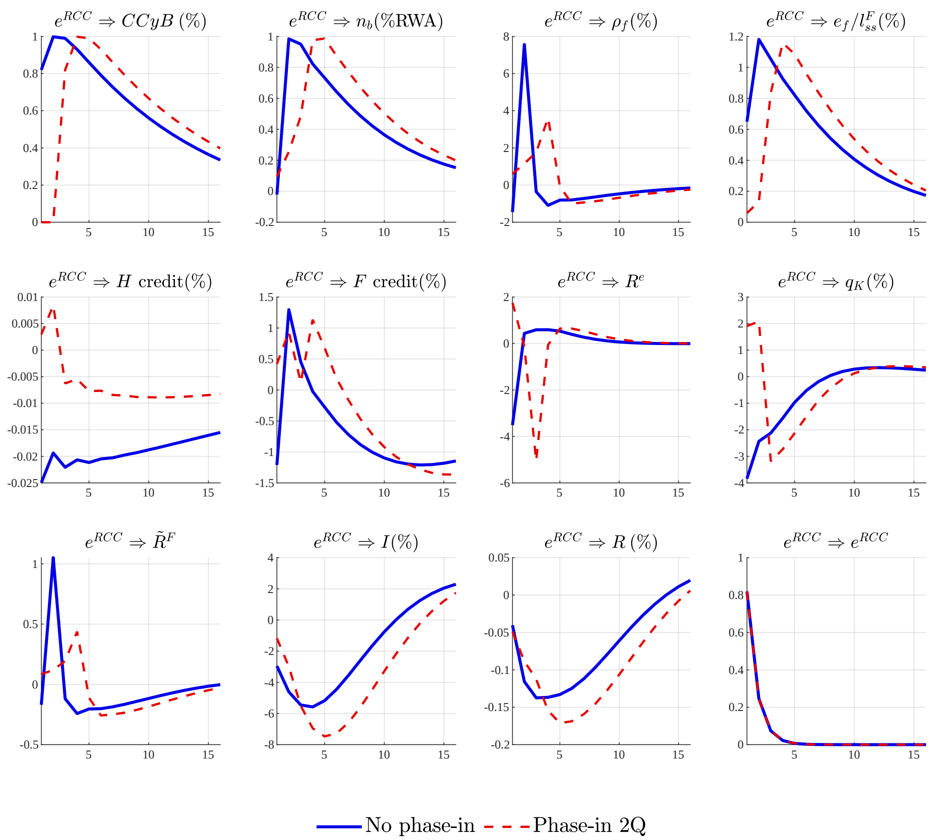
Financial Transmission Channel. Capital requirements are shocked such that the effective level of extra capital amounts to one percent of RWA in the period following the regulation shock.

On impact, total bankers' net worth cannot change, so effective capital in either bank branch can only be rebalanced in response to changes in expected relative profitability. Return on equity for commercial (short-term) credit becomes higher than the one for mortgage credit (long-term) inducing around a half-percentage point of capital to flow into the F-bank. On impact, however, commercial credit shrinks to accommodate the higher capital requirement. In the following period after impact, banks can meet the policy requirement by partly contracting loans or raising new capital. They respond by doing the latter. Capital prices decrease on impact and slowly recover, generating positive gross returns to projects by the entrepreneur and as a consequence for the commercial loan portfolio. In particular, loans in the second period expand rapidly, as recovery in capital prices imply a rebound on portfolio profitability. In later periods, as the CCyB requirement dissipates, bankers extra capital decreases in tandem with loans, but the latter diminish at a faster pace. This is reasonable as capital price changes are less dramatic the further the horizon, resulting in lower profitability of loans (see Figure 4).

A second effect of higher capital requirements comes from general equilibrium. Lower commercial loan activity implies weaker investment and lower inflation. Monetary policy rate responds accordingly and lowers the cost of short-term funding, further boosting lending.

Figure 3: Transmission Channel CCyB activation

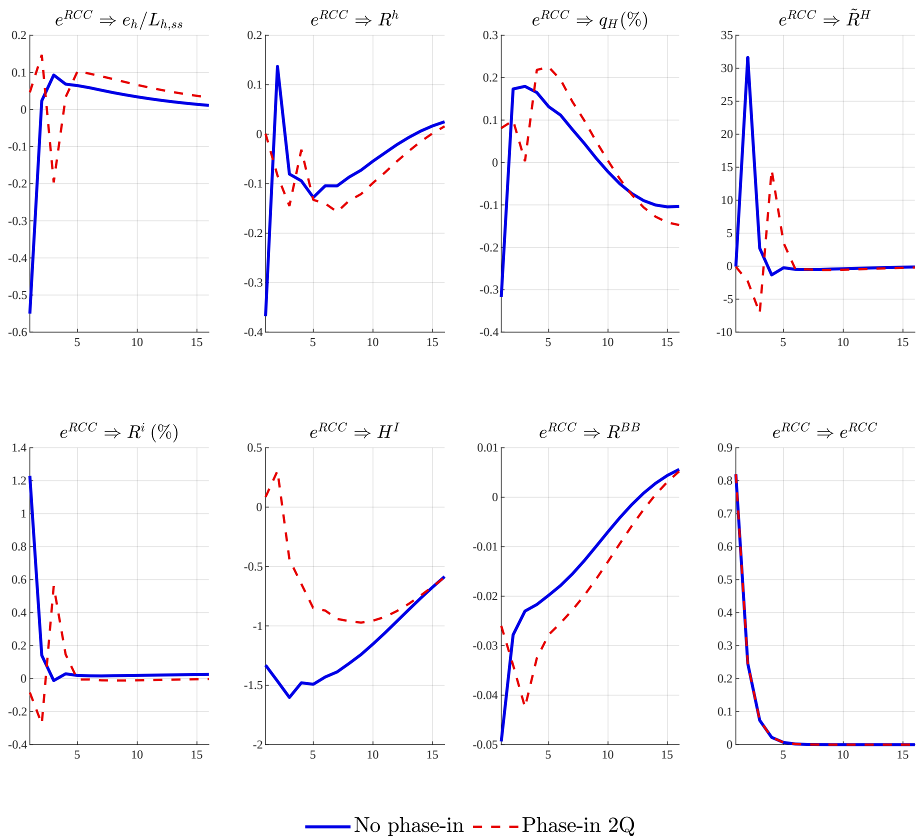
This figure shows the impulse response functions to an activation of CCyB of 100bp with no phase-in period (blue), which implies CCyB requirement must be met in full the period after its announcement, and the activation with a phase-in period of 2Q. Variables with (%) represent deviations from same variable steady states.



Moving to examine mortgage credit, we observe that on impact effective capital decreases. Because commercial credit is so much larger than mortgage credit, the decrease in capital to the H bank is more dramatic than the increase of capital in the F bank, even when their sum has not changed. Mortgage debt decreases, in spite of the duration of this type of debt and the fact that households face adjument costs to rebalance their demanded debt. The adjusting variable is its market price. In the periods after impact, there is more capital for both banks F and H, and as such the level of effective capital in bank H is larger than in steady state. However, the more stringent capital requirements are mostly met by decreasing the level of debt, which implies lower financing of impatient households housing stock, H^I, and higher interest rate on mortgage debt R^I

Figure 4: Transmission Channel CCyB activation II

This figure shows the impulse response functions to an activation of CCyB of 100bp with no phase-in period (blue), which implies CCyB requirement must be met in full the period after its announcement, and the activation with a phase-in period of 2Q. Variables with (%) represent deviations from same variable steady states.



The activation of the CCyB affects the rest of the economy too. The decline in the price of capital reduces net worth of entrepreneurs. Since the deposit rate declines in line with the policy rate, the marginal benefit of transferring a unit of resources to the future for the patient household λ_p decreases and consumption increases. For the impatient household the marginal benefit of transferring a unit of wealth into the future λ_i increases, according to equation (24), and consequently consumption decreases (Euler equation (16))

4.2 Simple Implementable Financial Policy Rules

One challenge in discussing the effects of financial policy rules is the lack of consensus on their structure. It is unclear what the neutral CCyB —the CCyB requirement when perceived systemic risk is moderate— level should be, and the literature has not established the minimum arguments on which changes of CCyB would depend. In this subsection, we examine various specifications for a financial policy rule and compare their effects on the long-term welfare of consumers. We focus exclusively on potential rules that may have an empirical counterpart, making them readily implementable. We refer to them as simple and implementable financial policy (SIFPR) rules.

Consumption equivalence. To find the optimal SIR we perform a welfare analysis in the spirit of Carrillo et al. (2021) and denote the welfare of the economy as $\mathbf{W}(\theta)$, using the equation (37) as follows

$$\mathbf{W}(\theta) = \sum_{i \in I,U,R} \wp_i \mathbb{E}_{0,i} \left\{ \sum_{t=1}^{\infty} \beta_i^t \varrho_t \left[\frac{1}{1-\sigma} \left(\hat{C}_t^i(\theta) \right)^{1-\sigma} - \Theta_t^i(\theta) A_t^{1-\sigma} \xi_t^n \frac{\left(n_t^i(\theta) \right)^{1+\varphi}}{1+\varphi} \right] \right\} \tag{28}$$

We calculate a baseline welfare $\mathbf{W^0} \equiv \mathbf{W}(\theta|\theta = 0)$ that summarizes consumers' welfare in an economy with no financial policy rule. This baseline statistic is useful for comparing gains or losses resulting from any CCyB rule activation under different specifications. Specifically, $\mathbf{W^0}$ is computed as the discounted value of the perpetual stream of constant period-utilities evaluated at the stochastic steady states of endogenous variables, $\hat{C}_{ss}^{i,\mathbf{0}}, \Theta_{ss}^{i,\mathbf{0}}, n_{ss}^{i,\mathbf{0}}$. We solve using a second-order perturbation and the pruning algorithm in Kim et al. (2008).

$$\mathbf{W^0} = \sum_{i \in I,U,R} \wp_i \frac{1}{1-\beta_i} \left[\frac{1}{1-\sigma} \left(\hat{C}_{ss}^{i,\mathbf{0}} \right)^{1-\sigma} - \Theta_{ss}^{i,\mathbf{0}} A_{ss}^{1-\sigma} \frac{\left(n_{ss}^{i,\mathbf{0}} \right)^{1+\varphi}}{1+\varphi} \right] \tag{29}$$

To more easily represent the gains from a given CCyB rule we compute consumption equivalent units, ce., This represents the permanent change in consumption that equates the welfare of the economy under a CCyB rule, $\mathbf{W}(\theta)$, and the welfare of the economy without a CCyB rule, $\mathbf{W^0}$. In other words, it is the level of permanent consumption required to offset the welfare gains/losses from implementing a certain rule ($\theta \neq 0$).

$$\mathbf{W}^{ce}(ce,\theta) = \sum_{i \in I,U,R} \wp_i \frac{1}{1-\beta_i} \left[\frac{1}{1-\sigma} \left(\hat{C}_{ss}^{i}(ce,\theta) \right)^{1-\sigma} - \Theta_{ss}^{i}(ce,\theta) A_{ss}^{1-\sigma} \frac{\left(n_{ss}^{i}(ce,\theta) \right)^{1+\varphi}}{1+\varphi} \right] = \mathbf{W^0} \tag{30}$$

where we adjust (36), (38), (39) accordingly,

$$\hat{C}_{ss}^i(ce,\theta) = \left[\left(1 - o_{\hat{C}}\right)^{\frac{1}{\eta_{\hat{C}}}} \left(C_{ss}^i (1-\phi_c)(1-ce) \right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}} + \left(o_{\hat{C}}\right)^{\frac{1}{\eta_{\hat{C}}}} \left(H_{ss}^i (1-\phi_{hh}) \right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}} \right]^{\frac{\eta_{\hat{C}}}{\eta_{\hat{C}}-1}} \tag{31}$$

$$\Theta_{ss}^i(ce,\theta) = \tilde{\chi}_{ss}^i(ce,\theta) A_{ss}^{\sigma} \left(\hat{C}_{ss}^i(ce,\theta) \right)^{-\sigma} \tag{32}$$

$$\tilde{\chi}_{ss}^i(ce,\theta) = A_{ss}^{-\sigma} \left(\hat{C}_{ss}^i(ce,\theta) \right)^{\sigma} \tag{33}$$

Therefore, when $ce > 0$ and $\mathbf{W^0} = \mathbf{W}^{ce}(ce,\theta)$ there is a welfare gain from implementing the SIR with respect to the baseline scenario. Consumers would need to reduce their consumption by $ce\%$ in order to be indifferent to living in the no-rule economy. Conversely, if $ce < 0$ then households are worse off because of the implementation of the CCyB rule, as they require a positive consumption wedge in order to be at least as good as in the no-rule economy.

Simple implementable financial policy rules (SIFPR). One of the challenges in assessing the marginal contribution of financial policy rules in models commonly used for monetary policy analysis, is the lack of consensus on how a Financial Policy Rule should look like. Our goal is to study rules which are implementable in the sense that they depend only on observable variables by policymakers, and are simple enough to guide expectation formation.

For monetary policy analysis, there is little disagreement around the most basic policy rule specification. The Taylor (1993) rule is not only a fair description of central banks' actual policy, but it also is a good approximation to the optimal Ramsey policy under fairly general assumptions (Woodford, 2001b). That said, the literature on optimal monetary policy has examined several variations and extensions to the Taylor rule, which is too vast to summarize here.

Counter-cyclical financial policy has become widespread after the global financial crisis, with emerging and advanced economies alike adopting the guidelines of Basel III. Yet we lack a consensus financial policy rule that guides financial policy as the Taylor rule guides monetary policy. Financial policy operates in most countries under "guided discretion" (ESRB, 2014), which combines the prescription of a mechanic rule with expert judgment nurtured by many financial risk indicators. The buffer guide most frequently suggested by the BCBS has been the credit-to-GDP gap rule (see BIS (2010); Drehmann (2013)). The logic behind this indicator is intuitive. Credit booms often precede financial stress. Raising buffers in booms and releasing them in busts helps stabilize the credit cycle and its amplification to real variables. However, there is little evidence that countries that activated the CCyB did so following the credit-to-GDP gap rule (see Herz and Keller (2023); Edge and Liang (2020)). Instead, activation has followed house price booms and the deterioration of banks' credit portfolios. This implies that in practice, national financial authorities have different assessments of financial policy rules. Our paper aims to inform on the quantitative properties of many options available to them, as long as they are simple (log-linear) and their inputs are observable to the policy maker.

Functional forms. We restrict to simple functional forms for the financial policy rule. In particular, we consider log-linear policy rules which are a function of the CCyB lag and an observable variable,

$$\left(\frac{1+CCyB_t}{1+\overline{CCyB}}\right) = \left(\frac{1+CCyB_{t-1}}{1+\overline{CCyB}}\right)^{\theta_1} \left(\frac{(1-\alpha_E)X_t + \alpha_E \mathbb{E}(X_{t+h})}{X}\right)^{\theta_2} e_t^{req} \tag{34}$$

In particular, if the neutral level CCyB is zero, the log-linear equivalent to (34) is

$$CCyB_t = \theta_1 CCyB_{t-1} + \theta_2 \log\left(\frac{(1-\alpha_E)X_t + \alpha_E \mathbb{E}X_{t+h}}{\overline{X}}\right) + \log(e^{req}) \tag{35}$$

We will consider X to represent:

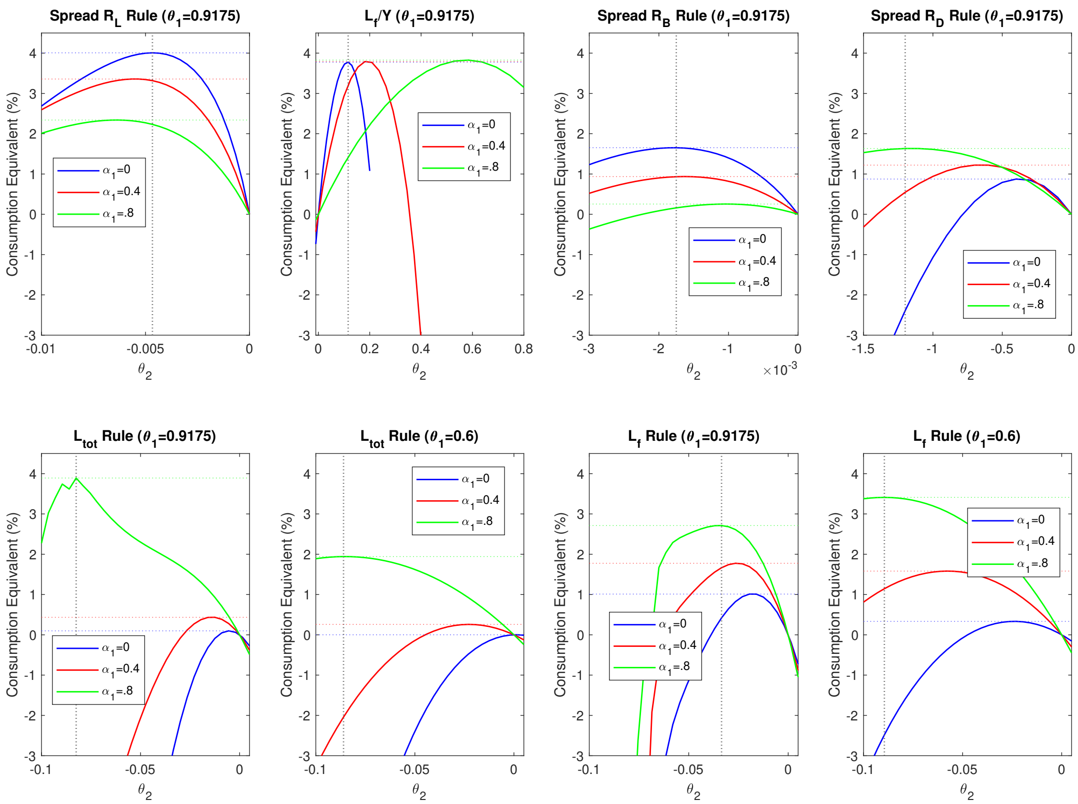
1. Commercial loan spread $R_t^L - R_t$ (the "Spread R_L rule"), an observable of the external finance premium as in Carrillo et al. (2021)
2. Credit-to-GDP (commercial credit) in the spirit of Drehmann (2013) and BIS (2010)
3. Spread $R_{B,t} - R_t$ with $R_{B,t}$ being the weighted average (by portfolio size) of commercial and mortgage rates (the "Spread R_B rule)
4. Spread $R_D - R$ (the "Spread R_D rule") to capture funding premium

5. Aggregate credit $L_{tot,t}$ (the "L_{tot} rule")

6. Commercial credit $L_{f,t}$ (the "L_f rule")

Figure 5: Consumption equivalence for different Rules

This figure shows the consumption equivalent for different values of θ_1 (controls inertia in countercylical capital requirements) and θ_2, the weight on the endogenous variable to which the rule reacts. Every sub-figure shows the results for different values of weight on expected variable X, α_E, and current realization of variable X.

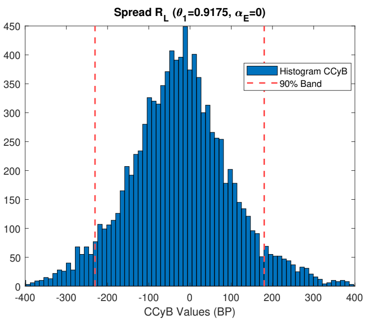


As we observe in the Figure 5 all the rules generates some welfare gains, measured as positive consumption equivalent, but at the same time, they can generate welfare loses for some values of θ_2 and α_E. This observation highlights the importance of choosing a sensible financial policy rule. We can see that the candidates to the best SIFPR are: (i) Spread R_L with $\alpha_E = 0$; (ii) Credit-to-GDP for all values of α_E; and (iii) L_{tot} with $\alpha_E = 0.8$ with a high degree of inertia for the CCyB ($\theta_1 = 0.9175$). All of them generate imply a 4% consumption equivalence.

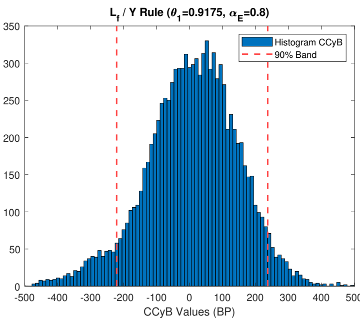
Importantly, however, we solve the model using perturbation around the steady state. This implies that for equation (34) we are not able take in to account that the countercyclical buffer goes from 0% to 2.5%. This implies that our welfare calculations reported in Figure (5) may be positively bias. We report the simulations of the CCyB for the three best performing rules in Figure 6, which shows the distribution of realizations of CCyB for the 10.000 simulations. We can clearly see that the L_{tot} rule is practically impossible to implemented since the 90% of CCyB values are between [-310,3173] bases points (see Figure 6c).

Figure 6: Distribution of Optimal CCyB simulated by rule.

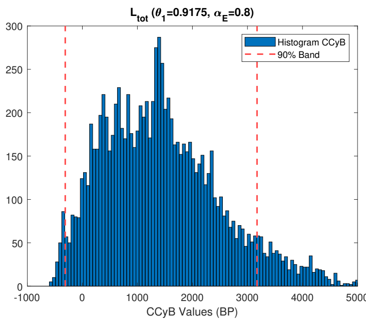
This figure shows the distribution of the CCyB simulations implied by the three best performing rules in terms of consumption equivalence. Each simulation consists of 10 thousand periods. Vertical axis shows frequency, the horizontal axis is expressed in basis points. The red dashed lines show the 90% interval calculated as the average $\pm 1.645\sigma_i$ with for each i rule.



(a) Spread R_L Rule.



(b) L_f/Y Rule.



(c) L_{tot} Rule.

Although the Spread R_L and L_f/Y rules do not generate extreme values for the CCyB, almost 70% of each values are outside the feasible region, in particular taking negative values. This goes in sharp contrast with implementation of the BIS (2010) principle of raising buffers when credit expands to lower it when the economy enters financial stress. In this business cycle model we see that two things happen instead. First, financial shock realizations can happen at any point without prior credit build-up. Second, it is often the case that the rule requires to lower the CCyB beyond prior accumulation, resulting in negative values. This observation on the implementation for the mechanic credit-to-GDP rule is consistent with many countries first considering to raise the buffer to achieve a "neutral" level, meaning a positive CCyB unrelated to shock realizations. The logic of a neutral level would be to have enough room to lower it without hitting the zero-lower bound implied by the design of the policy.

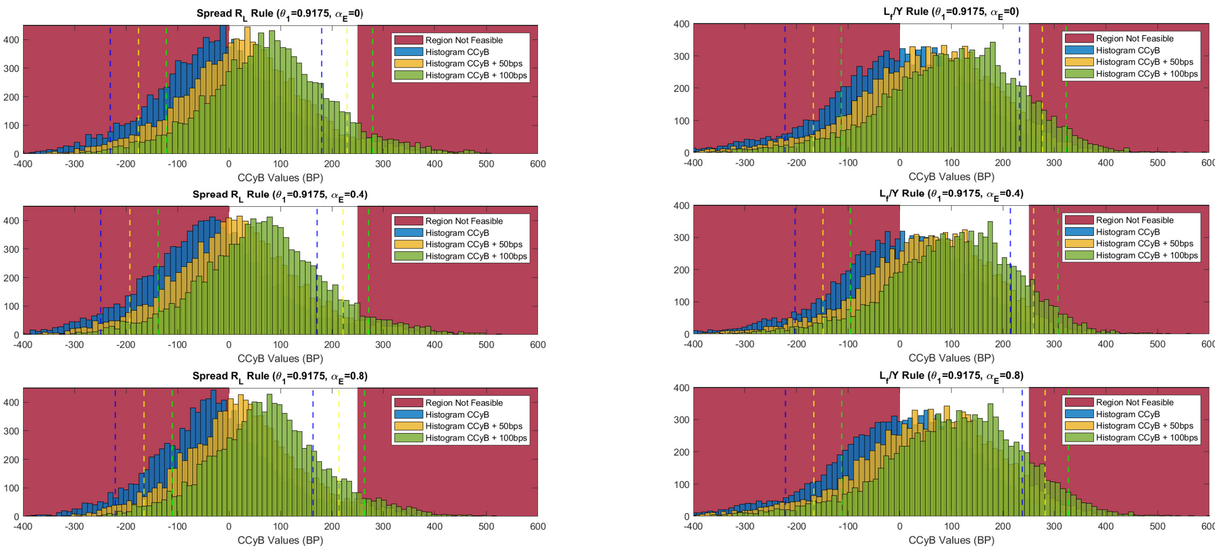
The next subsection we study the effects of include a neutral CCyB.

4.3 Optimal SIFPR with neutral CCyB

A positive neutral level for the CCyB, unrelated to shocks or endogenous variables, is understood in the model as steady-state capital, and it moves the distribution of the effective values of CCyB to the feasible region. The implementation of the CCyB is its infancy around the world, but initial evidence suggests that many countries have chosen to set a neutral level as their first policy action (see Herz and Keller, 2023). We study the effect of using a neutral CCyB level of 50 and 100 bps. The results are shown in the Figure 7.

Figure 7: Distribution of Optimal CCyB simulated with neutral CCyB.

This figure shows the distribution of the optimal CCyB simulations for **(a)** Spread R_L rule and **(b)** L_f/Y Rule. For both subfigures blue, yellow and green colors correspond to 0, 50 and 100 bps of neutral CCyB respectively. The dashed lines show the 90% interval calculated as the average $\pm 1.645 \cdot \sigma_i$ with i for each rule.



(a) Spread R_L Rule.

(b) L_f/Y Rule.

Specifically, Figure 7 shows the histogram for the CCyB simulations with the chosen rules and parameters found in the previous section. We distinguish three cases, denoted by colors blue, yellow and green histograms, corresponding to cases without neutral CCyB, with 50 and 100 basis points, respectively. For both rules, it can be seen that as the neutral CCyB increases, the part of the histogram in the feasible region increases.

Tables 6 and 7 summarize our findings in terms of feasibility and welfare gains. Welfare gains are calculated in terms of consumption equivalent defined in (30) and depicted in Figure 5. Feasibility is calculates as the probability that the CCyB simulations obtained for the rules are within the regulation boundaries of 0 and 250 basis points. Finally, the indicator that combines both criteria is expected equivalent consumption, defined as the product between feasibility and consumption equivalence. The latter statistic, is a first proxy that takes expected gains weighted by their implementation probability.

The first regularity in both rules (and tables) is the fact that consumption equivalence is decreasing in the level of neutral CCyB. For the spread rule we see that our previous 4% result in consumption equivalence can drop to 2.71% if the neutral CCyB level is 100bp. This is consistent with more capital in steady state reducing the level of default probability in steady states. If the economy is less vulnerable with the neutral level, it is less important for welfare to have a countercyclical rule. Also, this higher steady-state capital requirements reduce steady-state levels of investment, credit and ultimately output and consumption. Our statistic in tables 6 and 7 consider this effect.

The second regularity across rules (and tables) is that feasibility is non-monotone in the level of neutral CCyB. Higher neutral CCyB moves the distribution to the right. While 50bp increase the feasibility region from 38%

to 68% (50% to 66%) for the spread (loans-to-GDP) rule, further increasing it to 100bp actually decreases the frequency of realizations which are feasible. This is not surprising as the CCyB regulation has both an lower and an upper bound depicted in red in Figure 7.

Table 6: Spread R_L Rule Summary

This table summarizes the results of implementing the spread and the loan-to-GDP rules. In particular: (1) α_E denotes the forward looking component of the rule, (2) Permanent change in consumption that equates the welfare of the economy under a CCyB rule. (3) Probability that the simulations are in the feasible region of CCyB. Total number of times that the CCyB takes values between [0bp, 250bp] divided by the total number of simulations, (4) Expected Consumption Equivalent, calculated as the product between CE and the probability of feasibility.

CCyB Neutral (bps)	α_E[1]	Consumption equivalence (CE)[2]	Feasibility[3]	Expected CE[4]
0	0	4.01%	38.4%	1.57%
	0.4	3.36%	33.9%	1.14%
	0.8	2.34%	36.0%	0.84%
50	0	3.31%	68.1%	2.25%
	0.4	2.82%	64.4%	1.81%
	0.8	2.03%	68.8%	1.40%
100	0	2.71%	67.5%	1.82%
	0.4	2.34%	68.8%	1.60%
	0.8	1.73%	71.2%	1.23%

The third result we draw from our simulations is that the forward looking component of the rules, governed by parameter α^E, matters very little for the credit-to-GDP rule. This is in sharp contrast to the reasoning in Drehmann (2013) and BIS (2010), where the expected path of the credit cycle is a featured element, and also to any paralell we could draw from a Taylor rule for monetary policy. Even more, from Table 6, we see that the rules that do better are those that are less forward looking. The difference between a rule that has zero weight on future spread and a rule that weights future spreads with coefficient $\alpha^E = 0.8$ is more than 1% of CE. This result is consistent with the observation that financial shocks can be abrupt, and thus the target of the rule responds quickly to shocks and rules that offset these shocks just as quickly will perform better. This is not the case for demand shocks in setting monetary policy, because of sticky prices and nominal rigidities in general. Financial policy, then, is more effective when it is timely. This result further strengthen our argument for a neutral level of CCyB.

All previous elements considered, we find that the simple and implementable rule that performs best is one that responds to external finance premium –the Spread R_L– coupled with with 50 bps of neutral CCyB and no forward looking component. Interestingly, this is the same rule suggested in Carrillo et al. (2021) but further considers the impact of a zero lower bound and feasibility.

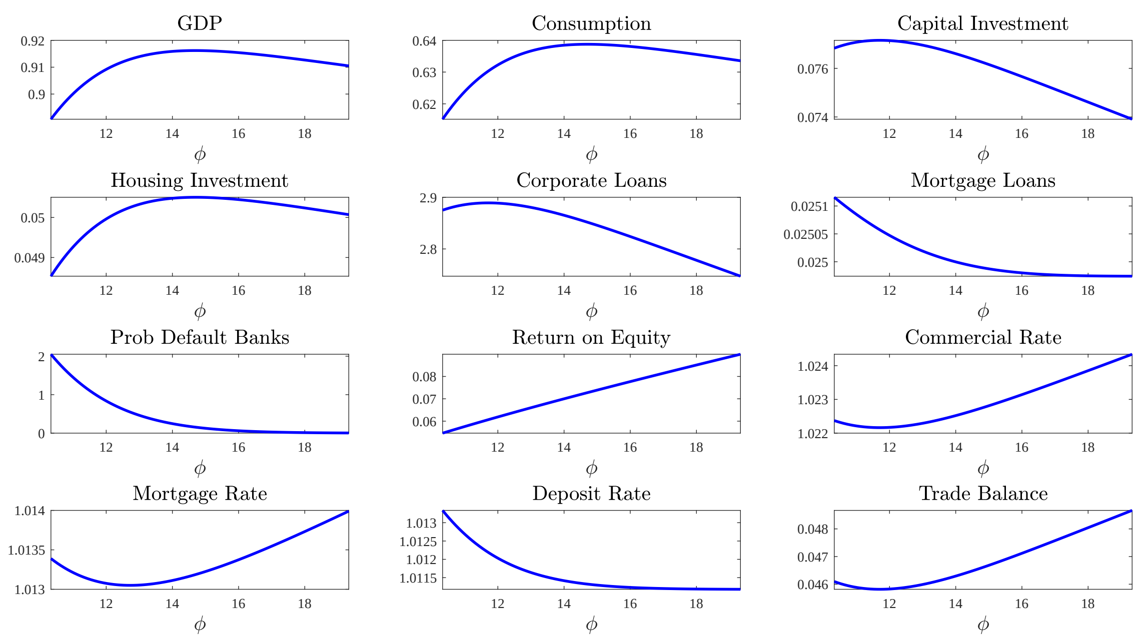
Table 7: L_f/Y Rule Summary

This table summarizes the results of implementing the spread and the loan-to-GDP rules. In particular: (1) α_E denotes the forward looking component of the rule, (2) Permanent change in consumption that equates the welfare of the economy under a CCyB rule. (3) Probability that the simulations are in the feasible region of CCyB. Total number of times that the CCyB takes values between [0bp, 250bp] divided by the total number of simulations, (4) Expected Consumption Equivalent, calculated as the product between CE and the probability of feasibility.

CCyB Neutral (bps)	α_E [1]	Consumption equivalence (CE) [2]	Feasibility[3]	Expected CE [4]
0	0	3.78%	50.7%	1.91%
	0.4	3.79%	51.8%	1.96%
	0.8	3.83%	51.1%	1.95%
50	0	2.79%	66.5%	1.85%
	0.4	2.79%	70.1%	2.01%
	0.8	2.83%	66.3%	1.87%
100	0	2.02%	54.5%	1.10%
	0.4	2.02%	56.6%	1.14%
	0.8	2.05%	53.8%	1.10%

Figure 8: Steady state for different capital requirements

This figure shows the steady state level for a set of endogenous variables in terms of different levels of bank capital. Bank capital in the horizontal axes (ϕ) includes the long-run average of voluntary buffers of 4.3%. This implies that neutral CCyB level of 0% is equivalent to $\phi = 14.05\%$, and that further increases of the neutral level operate in the decreasing part of the curve for consumption.



5 Conclusion

In this paper we have evaluated the welfare implications of introducing a countercyclical buffer rule which is simple and implementable. We do so by building a macro-banking model with two inefficiencies: nominal rigidities and financial frictions. This gives room for monetary and financial policies to be desirable. We use our model to study the functional form of a SIR for financial policy. Further, we argue that the countercyclical nature of the CCyB and its institutional design (zero lower bound) imply a rationale for a neutral positive level of the buffer.

Using a quantitative model estimated with Chilean data, we explore several simple and implementable financial policy rules in terms of welfare differences (summarized in consumption equivalent terms). We find that, consumption equivalence is decreasing in the level of neutral CCyB, the lower and upper limits of 0 and 250bp are binding, and that rules put weight on future expected realizations of endogenous variables perform no better than rules that respond quickly to shocks. This goes in sharp contrast to conventional wisdom and efforts to forecast the financial cycle as a useful indicator for setting the CCyB.

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ONLINE APPENDIX

A Full Model Details

A.1 Households

There are two continuums of households, each of measure one, risk-averse and infinitely lived, impatient (I) and patient (P) with discount factors β_I and $\beta_P > \beta_I$, respectively. In turn, patient households can be restricted (R) and unrestricted (U) depending on which assets they can access to save. Unrestricted households can buy both long- and short-term assets with a transaction cost, Restricted households can only buy long-term bonds but do not face any transaction cost. Their combined measure is of size one. This segmentation follows Andres et al. (2004) and Chen et al. (2012).

Restricted and Unrestricted households' preferences depend on consumption of a final good C_t relative to external habits $\tilde{C}_{t-1}$, their stock of housing from last period H_{t-1} relative to external habits $\tilde{H}_{t-2}$, and labor supplied (hours worked) n_t in each period. The consumption of the aggregate good $\hat{C}^i_t \equiv \hat{C}(C^i_t, \tilde{C}^i_{t-1}, H^i_{t-1}, \tilde{H}^i_{t-2})$ for households of type $i = \{U, R, I\}$ is assumed to be a constant elasticity of substitution (CES) as shown in (36):

$$\hat{C}^i_t = \left[\left(1 - o_{\hat{C}}\right)^{\frac{1}{\eta_{\hat{C}}}} \left(C^i_t - \phi_c \tilde{C}^i_{t-1}\right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}} + \left(o_{\hat{C}}\right)^{\frac{1}{\eta_{\hat{C}}}} \left(\xi^h_t \left(H^i_{t-1} - \phi_{hh} \tilde{H}^i_{t-2}\right)\right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}} \right]^{\frac{\eta_{\hat{C}}}{\eta_{\hat{C}}-1}} \tag{36}$$

where $o_{\tilde{C}} \in (0,1)$ is the weight on housing in the aggregate consumption basket, $\eta_{\tilde{C}}$ is the elasticity of substitution between the final good and the housing good, ξ^h_t is an exogenous preference shifter shock and $\phi_c, \phi_{hh} \geq 0$ are parameters guiding the strength of external habits in consumption and housing respectively. Households of type i maximize the following expected utility

$$\max_{\{\hat{C}^i_t, H^i_t\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=1}^{\infty} \beta^t_i \varrho_t \left[\frac{1}{1-\sigma} \left(\hat{C}^i_t\right)^{1-\sigma} - \Theta^i_t A^{1-\sigma}_t \xi^n_t \frac{\left(n^i_t\right)^{1+\varphi}}{1+\varphi} \right] \tag{37}$$

where $\beta_i \in (0,1)$ is the respective discount factor, ϱ_t is an exogenous shock to intertemporal preferences, ξ^n_t is a preference shock that affects the (dis)utility from labor, $\sigma > 0$ is the inverse of the intertemporal elasticity of substitution, $\varphi \geq 0$ is the inverse elasticity of labor supply. As in Galí et al. (2012), we introduce an endogenous preference shifter Θ_t, that satisfies the following conditions

$$\Theta^i_t = \tilde{\chi}^i_t A^{\sigma}_t \left(\hat{C}\left(\tilde{C}^i_t, \tilde{C}^i_{t-1}, \tilde{H}^i_{t-1}, \tilde{H}^i_{t-2}\right)\right)^{-\sigma} \tag{38}$$

and

$$\tilde{\chi}^i_t = \left(\tilde{\chi}^i_{t-1}\right)^{1-v} A^{-\sigma v}_t \left(\hat{C}\left(\tilde{C}^i_t, \tilde{C}^i_{t-1}, \tilde{H}^i_{t-1}, \tilde{H}^i_{t-2}\right)\right)^{\sigma v} \tag{39}$$

where the parameter $v \in [0,1]$ regulates the strength of the wealth effect, and $\tilde{C}^i_t$ and $\tilde{H}^i_{t-1}$ are taken as given by the households. In equilibrium $C^i_t = \tilde{C}^i_t$ and $H^i_t = \tilde{H}^i_t$.

A.1.1 Patient Households

Recall Patient households can be either restricted or unrestricted in terms of the assets they have access to.

Unrestricted Households. This group is formed by fraction $\wp_U$ of the patient households. They save in one-period government bonds (BS^U_t), long-term government bonds (BL^U_t), short-term bank deposits (D^U_t), long-term bank-issued bonds (BB^U_t), and one-period foreign bonds quoted in foreign currency $B^{\star U}_t$. All non-state-contingent assets.

Following Woodford (2001a), long-term instruments are perpetuities, paying a coupon of one unit of final good in the period after origination, and ($\kappa < 1$) geometrically declining coupons thereafter. Let B_{t-1} represents total liabilities due in t,

$$B_{t-1} = CI_{t-1} + \kappa CI_{t-2} + \kappa^2 CI_{t-3} + \ldots,$$

then, $CI_{t-1} = B_{t-1} - \kappa B_{t-2}$. Also, let Q_t^B denote the period-t price of a new bond, then Q_t^B summarizes the prices at all maturities. For instance, $Q_{t|t-1}^B = \kappa Q_t^B$ is the price in t of a perpetuity issued in period $t-1$. Importantly, note that B_{t-1} coincides with the total number of outstanding bonds. Then, the total value of financial asset debt in period t is given by $Q_t B_t$. Finally, the *real* yield to maturity of holding long term assets at period t, R_t^B, as,

$$R_t^B = \frac{P_t}{Q_t^B} + \kappa$$

Unrestricted households must pay a transaction cost ζ_t^L per unit of long-term bond purchased. These costs are paid to a financial intermediary as a fee. This financial intermediary distributes its nominal value profits Π^{FI}, as dividends to its shareholders. Then, unrestricted patient households' period budget constraint equates uses and sources of funds,

$$\begin{aligned} BS_t^U + D_t^U + S_t B_t^{\star U} + \left(1 + \zeta_t^L\right)\left(Q_t^{BL} BL_t^U + Q_t^{BB} BB_t^U\right) + P_t C_t^U + Q_t^H \left(H_t^U - (1-\delta_H) H_{t-1}^U\right) = \\ R_{t-1} BS_{t-1}^U + Q_t^{BL} R_t^{BL} BL_{t-1}^U + \tilde{R}_t^D D_{t-1}^U + \tilde{R}_t^{BB} Q_t^{BB} BB_{t-1}^U + S_t B_{t-1}^{\star U} R_{t-1}^{\star} + W_t n_t^U + \Psi_t \end{aligned} \tag{40}$$

where R_t^{BL} and R_t^{BB} are the real gross yield to maturity for long-term government and bank-issued bonds at time t, P_t denotes the price of the consumption good, Q_t^H denotes the nominal price of housing good, δ_H is depreciation rate of housing goods, S_t the nominal exchange rate (units of domestic currency per unit of foreign currency), $R_t^{\star}$ the foreign one-period bond return, and R_t denotes the short term nominal government bond rate.

Further, $\widetilde{R}_t^D = R_{t-1}^D (1 - \gamma_D PD_t^B)$, $\widetilde{R}_t^{BB} = R_t^{BB}(1 - \gamma_{BB} PD_t^B)$ denote the net return on deposits and net yield on bank-bonds, received by households. Also, R_{t-1}^D is the gross interest rate paid by banks in t, PD_t^B denotes the default probability of banks, and γ_D and γ_{BB}) denote transaction costs that households must pay in order to recover their funds, even under deposit insurance. Finally, W_t denotes the nominal wage and, Ψ_t denotes lump-sum payments that include taxes T_t, dividend income from entrepreneurs C_t^e, bankers C_t^b, rents from ownership of foreign firms REN_t^*, profits from ownership of domestic firms, and profits from the financial intermediary in the long-term bond transactions, $\Pi^F = \zeta_t^L (Q_t^{BL} BL_t^U + Q_t^{BB} BB_t^U)$.

We assume that ζ_t^L is a function of the ratio of the long- to short-term assets held by the unrestricted agent, plus a disturbance term ϵ_t^L. Households do not internalize the effect of their choices on this transaction cost, yet in equilibrium $\widetilde{BL}_t^U = BL_t^U$, $\widetilde{BS}_t^U = BS_t^U$ and the discounted value of future transaction costs implies a term premium (Chen et al., 2012),

$$\zeta_t^L = \left(\frac{Q_t^{BL} \widetilde{BL}_t^U + Q_t^{BB} \widetilde{BB}_t^U}{\overline{Q}_t^{BL} \overline{BL}_t^U + \overline{Q}_t^{BB} \overline{BB}_t^U} \right)^{\eta_{\zeta_L}} \epsilon_t^L \tag{41}$$

Households supply differentiated labor services to a continuum of *unions* which act as wage setters on behalf of the households in monopolistically competitive markets. The unions collect the wage income from all households and distribute it equally among them, providing insurance against wage-income risk. Defining for convenience the multiplier on the budget constraint as $\lambda_t^U A_t^{-\sigma}/P_t$, then, Unrestricted Households solve (37) subject to (36), (38), (39), and (40). From this problem, we obtain the following first-order conditions:

$$[C_t^U]: \quad \lambda_t^U A_t^{-\sigma} = \left(\hat{C}_t^U\right)^{-\sigma} \left(\frac{(1-o_{\hat{C}})\,\hat{C}_t^U}{\left(C_t^U - \phi_c \tilde{C}_{t-1}^U\right)} \right)^{\frac{1}{\eta_{\hat{C}}}} \tag{42}$$

$$\begin{aligned}[H_t^P]: \quad \varrho_t \frac{\lambda_t^U A_t^{-\sigma} Q_t^H}{P_t} =& \beta_U \mathbb{E}_t \varrho_{t+1} \Bigg\{ \left(\hat{C}_{t+1}^U\right)^{-\sigma} \xi_{t+1}^h \left(\frac{o_{\hat{C}} \hat{C}_{t+1}^U}{\xi_{t+1}^h \left(H_t^U - \phi_{hh} \tilde{H}_{t-1}^U\right)} \right)^{\frac{1}{\eta_{\hat{C}}}} \\ &+ (1-\delta_H) \frac{\lambda_{t+1}^U A_{t+1}^{-\sigma} Q_{t+1}^H}{P_{t+1}} \Bigg\}\end{aligned} \tag{43}$$

$$[BS_t^U]: \quad \varrho_t \lambda_t^U A_t^{-\sigma} = \beta_U R_t \mathbb{E}_t \left\{ \frac{\varrho_{t+1} \lambda_{t+1}^U}{\pi_{t+1}} A_{t+1}^{-\sigma} \right\} \tag{44}$$

$$[BL_t^U]: \quad \varrho_t \lambda_t^U A_t^{-\sigma} (1+\zeta_t^L) \left(\frac{Q_t^{BL}}{P_t} \right) = \beta_U \mathbb{E}_t \left\{ \varrho_{t+1} \lambda_{t+1}^U A_{t+1}^{-\sigma} R_{t+1}^{BL} \left(\frac{Q_{t+1}^{BL}}{P_{t+1}} \right) \right\} \tag{45}$$

$$[B_t^{\star U}]: \quad \varrho_t \lambda_t^U A_t^{-\sigma} = \beta_U R_t^{\star} \mathbb{E}_t \left\{ \frac{\varrho_{t+1} \lambda_{t+1}^U \pi_{t+1}^s}{\pi_{t+1}} A_{t+1}^{-\sigma} \right\} \tag{46}$$

$$[D_t^U]: \quad \varrho_t \lambda_t^U A_t^{-\sigma} = \beta_U \mathbb{E}_t \left\{ \frac{\varrho_{t+1} \lambda_{t+1}^U}{\pi_{t+1}} \tilde{R}_{t+1}^D A_{t+1}^{-\sigma} \right\} \tag{47}$$

$$[BB_t^U]: \quad \varrho_t \lambda_t^U A_t^{-\sigma} (1+\zeta_t^L) \left(\frac{Q_t^{BB}}{P_t} \right) = \beta_U \mathbb{E}_t \left\{ \varrho_{t+1} \lambda_{t+1}^U A_{t+1}^{-\sigma} \tilde{R}_{t+1}^{BB} \left(\frac{Q_{t+1}^{BB}}{P_{t+1}} \right) \right\} \tag{48}$$

In equilibrium, we have that $\tilde{C}_t^P = C_t^P$ and $\tilde{H}_t^P = H_t^P$.

Restricted households. The main difference with Unrestricted households is that they can only access long-term government bonds, and do not face transaction cost ζ_t^L. Their mass is $\wp_R$, and their period BC is

$$P_t C_t^R + Q_t^H \left(H_t^R - (1-\delta_H) H_{t-1}^R \right) + Q_t^{BL} BL_t^R = W_t n_t^R + Q_t^{BL} R_t^{BL} BL_{t-1}^R \tag{49}$$

For convenience, let the multiplier on the budget constraint be $\lambda_t^R A_t^{-\sigma}/P_t$. Then, restricted households solve (37) subject to (36), (38), (39), and (49), from which we obtain the following first-order conditions:

$$[C_t^R]: \quad \lambda_t^R A_t^{-\sigma} = \left(\hat{C}_t^R\right)^{-\sigma} \left(\frac{(1-o_{\hat{C}})\,\hat{C}_t^R}{\left(C_t^R - \phi_c \tilde{C}_{t-1}^R\right)} \right)^{\frac{1}{\eta_{\hat{C}}}} \tag{50}$$

$$\begin{aligned}[H_t^P]: \quad \varrho_t \frac{\lambda_t^R A_t^{-\sigma} Q_t^H}{P_t} =& \beta_R \mathbb{E}_t \varrho_{t+1} \Bigg\{ \left(\hat{C}_{t+1}^R\right)^{-\sigma} \left(\frac{o_{\hat{C}} \hat{C}_{t+1}^R}{\xi_{t+1}^h \left(H_t^R - \phi_{hh} \tilde{H}_{t-1}^R\right)} \right)^{\frac{1}{\eta_{\hat{C}}}} \xi_{t+1}^h \\ &+ (1-\delta_H) \frac{\lambda_{t+1}^R A_{t+1}^{-\sigma} Q_{t+1}^H}{P_{t+1}} \Bigg\}\end{aligned} \tag{51}$$

$$[BL_t^R]: \quad \varrho_t \lambda_t^R A_t^{-\sigma} Q_t^{BL} = \beta_R \mathbb{E}_t \left\{ \frac{\varrho_{t+1} \lambda_{t+1}^R}{\pi_{t+1}} R_{t+1}^{BL} Q_{t+1}^{BL} A_{t+1}^{-\sigma} \right\} \tag{52}$$

A.1.2 Impatient Households

Impatient households work, consume, and purchase housing goods. They borrow long-term to finance their purchases of housing. The implicit real yield to maturity on mortgage debt R_t^I at date-t is,

$$R_t^I = \left(\frac{P_t}{Q_t^L} + \kappa \right),$$

where Q_t^L is the price of one unit of long-term mortgage debt L_t^H issued in period-t, and κ is the geometric decline factor of long-term debt, as in the case of government debt.

In any time period after t, banks and households adhere to the initial contract agreement. This allows us to account for the fact that, when it comes to default decisions, households are more concerned about the face value of their debt rather than the market value. This is a more accurate portrayal of the fixed-condition Chilean mortgage market. Then, the nominal face value of mortgage credit $L_t^H \widehat{Q}_t^L$, is the sum of newly issued debt priced at current market conditions, and debt from previous periods priced issue-date market conditions,

$$L_t^H \widehat{Q}_t^L = (L_t^H - \kappa L_{t-1}^H) Q_t^L + \kappa L_{t-1}^H \widehat{Q}_{t-1}^L \pi_t \tag{53}$$

A second reason for tracking face-value of the mortgage portfolio is that this is the actual time series we observe in the data and use to estimate the model.

Mortgage Default. We assume mortgage loans are non-recourse, limited liability contracts, which makes default an option for households. Households default when period liabilities $(P_t + \kappa \widehat{Q}_{t-1}^L \pi_t) L_{t-1}^H$ exceed the value of the assets used as collateral, $\omega_t^I Q_t^H (1-\delta_H) H_{t-1}^I$, or

$$\widehat{R}_t^I \widehat{Q}_t^L L_{t-1}^H > \omega_t^I R_t^H Q_{t-1}^H H_{t-1}^I \tag{54}$$

where we have used $\widehat{R}_t^I = \frac{P_t + \kappa \widehat{Q}_{t-1}^L \pi_t}{\widehat{Q}_t^L}$, $R_t^H = \frac{Q_t^H (1-\delta_H)}{Q_{t-1}^H}$, and introduced ω_t^I as an i.i.d idiosyncratic shock to the efficiency units of housing of impatient households, which follows a log-normal distribution with pdf $f_I\left(\omega_t^I\right)$ and cdf $F_I\left(\omega_t^I\right)$, and can be interpreted as a reduced-form representation of any shock to the value of houses. Then, the default threshold $\bar{\omega}_t^I$ is given by

$$\bar{\omega}_t^I = \frac{\widehat{R}_t^I \widehat{Q}_t^L L_{t-1}^H}{R_t^H Q_{t-1}^H H_{t-1}^I}$$

If $\omega_t^I \geq \bar{\omega}_t^I$, the impatient household remains in good standing and repays the amount $\widehat{R}_t^I \widehat{Q}_t^L L_{t-1}^H$, else, the household defaults on its mortgage debt. This definition allows us to define $PD_t^I = F_I\left(\bar{\omega}_t^I\right)$ as the default rate of mortgages. In case of repayment, the bank receives the fixed amount $\widehat{R}_t^I \widehat{Q}_t^L L_{t-1}^H$ from performing loans, and households walk away with $(\omega_t^I - \bar{\omega}_t^I) R_t^H Q_{t-1}^H H_{t-1}^I$. In case of default the bank recovers $(1-\mu_I)\omega_t^I R_t^H Q_{t-1}^H H_{t-1}^I$ and the household walks away with nothing. This mechanism, a standard debt contract, is not only incentive-compatible on the side of the bank but also induces truth-telling on the side of the household.

Budget constraint. The budget constraint for the impatient household equates uses and sources of funds,

$$\begin{aligned} P_t C_t^I + Q_t^H H_t^I - Q_t^L (L_t^H - \kappa L_{t-1}^H) \left[1 - \frac{\gamma_L}{2}\left(\frac{L_t^H - \kappa L_{t-1}^H}{L_{t-1}^H - \kappa L_{t-2}^H} - \bar{a}\right)^2\right] - \pi_t \kappa L_{t-1}^H \widehat{Q}_{t-1}^L = \\ W_t n_t^I + \int_0^\infty \max\left\{\omega_t^I R_t^H Q_{t-1}^H H_{t-1}^I - \widehat{R}_t^I \widehat{Q}_t^L L_{t-1}^H, 0\right\} dF_I(\omega_t^I) \end{aligned} \tag{55}$$

where the expression $\left[1 - \frac{\gamma_L}{2}\left(\frac{L_t^H - \kappa L_{t-1}^H}{L_{t-1}^H - \kappa L_{t-2}^H} - a_t\right)^2\right]$ represents the adjustment costs to changing the level of debt L_t^H. The second term in the RHS captures the default decision.

Following Bernanke et al. (1999), the share of the gross return of financing housing, that goes to the bank is denoted by $\Gamma_I(\bar{\omega}_t^I)$. The rest goes to the household, where:

$$\Gamma_I\left(\bar{\omega}_t^I\right) = \int_0^{\bar{\omega}_t^I} \omega_t^I f_I\left(\omega_t^I\right) d\omega_t^I + \bar{\omega}_t^I \int_{\bar{\omega}_t^I}^\infty f_I\left(\omega_t^I\right) d\omega_t^I$$

This allows us to rewrite the budget condition as

$$P_t C_t^I + Q_t^H H_t^I - Q_t^L (L_t^H - \kappa L_{t-1}^H) \left[1 - \frac{\gamma_L}{2}\left(\frac{L_t^H - \kappa L_{t-1}^H}{L_{t-1}^H - \kappa L_{t-2}^H} - \bar{a}\right)^2\right] - \pi_t \kappa L_{t-1}^H \widehat{Q}_{t-1}^L = W_t n_t^I + \left[1 - \Gamma_I\left(\bar{\omega}_t^I\right)\right] R_t^H Q_{t-1}^H H_{t-1}^I \tag{56}$$

Also, let $G_I\left(\bar{\omega}_t^I\right) = \int_0^{\bar{\omega}_t^I} \omega_t^I f_I\left(\omega_t^I\right) d\omega_t^I$ denote the part of those returns that comes from the defaulted loans. Taking into consideration the share of the return that is lost due to verification cost as $\mu_I G_I(\bar{\omega}_t^I)$, then the net share of return that goes to the bank is $\Gamma_I\left(\bar{\omega}^I_t\right) - \mu_I G_I\left(\bar{\omega}^I_t\right)$. The participation constraint of the bank is

$$\mathbb{E}_t\left\{\left[1-\Gamma^H\left(\bar{\omega}^H_{t+1}\right)\right]\left[\Gamma_I\left(\bar{\omega}^I_{t+1}\right)-\mu_I G_I\left(\bar{\omega}^I_{t+1}\right)\right] R^H_{t+1} Q^H_t H^I_t\right\} \geq \rho^H_{t+1}\phi_H Q^L_t L^H_t \tag{57}$$

Where $1-\Gamma^H(\bar{\omega}^H_{t+1})$ is the fraction of gross returns from the bank mortgage portfolio left to bank-equity owners (more detail below). The rest of the LHS expression is the return on the housing project that goes to the lender bank. The RHS indicates the opportunity cost, which is investing an amount of equity $\phi_H Q^L_t L^H_t$ at a market-determined rate of return of $\tilde{\rho}^H_{t+1}$, where ϕ_H is a regulatory capital constraint. We elaborate on the bank's problem in subsection A.3, for now, note that (57) hold in equilibrium.

Thus, following the timing described above, the impatient household's optimization problem can be written as maximizing (37) for $i = I$ subject to their budget constraint (56) and the bank participation constraint (57). For convenience define $\lambda^I_t A_t^{-\sigma}/P_t$ and $\lambda^H_t A_t^{-\sigma}/P_t$ as the multipliers for each constraint respectively. This yields the following FOCs:

$$[C^I_t]: \qquad \lambda^I_t A_t^{-\sigma} = \left\{\left(\hat{C}^I_t\right)^{-\sigma}\right\}\left(\frac{\left(1-o_{\hat{C}}\right)\hat{C}^I_t}{\left(C^I_t - \phi_c \tilde{C}^I_{t-1}\right)}\right)^{\frac{1}{\eta_{\hat{C}}}} \tag{58}$$

$$[H^I_t]: \qquad \varrho_t\frac{\lambda^I_t A_t^{-\sigma} Q^H_t}{P_t} = \mathbb{E}_t\left\{\begin{array}{l} \beta_I\varrho_{t+1}\left(\left(\hat{C}^I_{t+1}\right)^{-\sigma}\left(\frac{o_{\hat{C}}\hat{C}^I_{t+1}}{\xi^h_{t+1}\left(H^I_t-\phi_{hh}\tilde{H}^I_{t-1}\right)}\right)^{\frac{1}{\eta_{\hat{C}}}}\xi^h_{t+1}\right. \\ \left. +\frac{\lambda^I_{t+1}A_{t+1}^{-\sigma}}{P_{t+1}}\left[1-\Gamma_I\left(\bar{\omega}^I_{t+1}\right)\right]R^H_{t+1}Q^H_t\right) \\ +\frac{\varrho_t\lambda^H_t A_t^{-\sigma}}{P_t}\left[1-\Gamma^H\left(\bar{\omega}^H_{t+1}\right)\right]\left[\Gamma_I\left(\bar{\omega}^I_{t+1}\right)-\mu_I G_I\left(\bar{\omega}^I_{t+1}\right)\right]R^H_{t+1}Q^H_t \end{array}\right\} \tag{59}$$

$$\begin{aligned}[L^H_t]: \qquad & \varrho_t\frac{A_t^{-\sigma}Q^L_t}{P_t}\left\{\lambda^I_t\left[1-\frac{\gamma_L}{2}(\nabla\tilde{l}_t-\bar{a})^2\right]-\lambda^I_t\nabla\tilde{l}_t\gamma_L(\nabla\tilde{l}_t-\bar{a})-\lambda^H_t\rho^H_{t+1}\phi_H\right\} = \ldots \\ & \beta_I\mathbb{E}_t\left\{\varrho_{t+1}\frac{\lambda^I_{t+1}A_{t+1}^{-\sigma}}{P_{t+1}}\left[Q^L_{t+1}\kappa\left[1-\frac{\gamma_L}{2}(\nabla\tilde{l}_{t+1}-\bar{a})^2\right]\right]\right\}+ \\ & \beta_I\mathbb{E}_t\left\{\varrho_{t+1}\frac{\lambda^I_{t+1}A_{t+1}^{-\sigma}}{P_{t+1}}\left[-Q^L_{t+1}\nabla\tilde{l}_{t+1}\gamma_L(\nabla\tilde{l}_{t+1}-\bar{a})(\nabla\tilde{l}_{t+1}+\kappa)-\kappa\pi_{t+1}\hat{Q}^L_t\right]\right\}\end{aligned} \tag{60}$$

$$[\omega^I_t]: \qquad \frac{\varrho_t\lambda^H_t A_t^{-\sigma}}{P_t}\mathbb{E}_t\left\{\left[1-\Gamma^H\left(\bar{\omega}^H_{t+1}\right)\right]\left[\Gamma'_I\left(\bar{\omega}^I_{t+1}\right)-\mu_I G'_I\left(\bar{\omega}^I_{t+1}\right)\right]\right\} = \beta_I\mathbb{E}_t\left\{\frac{\varrho_{t+1}\lambda^I_{t+1}A_{t+1}^{-\sigma}}{P_{t+1}}\Gamma'_I\left(\bar{\omega}^I_{t+1}\right)\right\} \tag{61}$$

Functional forms idiosyncratic shocks ω . We draw from Bernanke et al. (1999) and assume that $\ln(\omega^I_t) \sim N(-\frac{1}{2}(\sigma^I_t)^2, (\sigma^I_t)^2)$, therefore its unconditional expectation is $\mathbb{E}\{\omega^I_t\} = 1$, and its average conditional on truncation is

$$\mathbb{E}_t\left\{\omega^I_t|\omega^I_t \geq \bar{\omega}^I_t\right\} = \frac{1-\Phi\left(z^I_t-\sigma^I_t\right)}{1-\Phi\left(z^I_t\right)},$$

where Φ is the c.d.f. of the standard normal and z^I_t is an auxiliary variable defined as $z^I_t \equiv (\ln(\bar{\omega}^I_t)+0.5(\sigma^I_t)^2)/\sigma^I_t$. Then, we can obtain the following functional forms:

$$\Gamma_I\left(\bar{\omega}^I_t\right) = \Phi\left(z^I_t-\sigma^I_t\right)+\bar{\omega}^I_t\left(1-\Phi\left(z^I_t\right)\right)$$

and

$$\Gamma_I\left(\bar{\omega}^I_t\right)-\mu_I G_I\left(\bar{\omega}^I_t\right) = \left(1-\mu_I\right)\Phi\left(z^I_t-\sigma^I_t\right)+\bar{\omega}^I_t\left(1-\Phi\left(z^I_t\right)\right)$$

Finally, we allow for fluctuations in the variance of the idiosyncratic shock σ^I_t, as in Christiano et al. (2014) and Carrillo et al. (2021).

A.2 Entrepreneurs

As in Clerc et al. (2014), we introduce risk-neutral entrepreneurs that follow an overlapping generations structure, where each generation lives across two consecutive periods. The entrepreneurs are the sole owners of productive capital, which is bought from capital producers to be, in turn, rented to the firms that produce different varieties of the home good.

Entrepreneurs born in period t draw utility in $t+1$ from transferring part of final wealth as dividends, C^e_{t+1}, to unrestricted patient households and from leaving the rest as bequests, N^e_{t+1}, to the next generation of entrepreneurs in the form:

$$\max_{C^e_{t+1},N^e_{t+1}} \left(C^e_{t+1}\right)^{\xi_{\chi_e}\chi_e}\left(N^e_{t+1}\right)^{1-\xi_{\chi_e}\chi_e} \text{ subject to}$$

$$C^e_{t+1}+N^e_{t+1}=\Psi^e_{t+1}$$

where Ψ^e_{t+1} is entrepreneurial wealth at $t+1$, explained below, and ξ_{χ_e} is a stochastic shock to their preferences. The first order conditions to this problem may be written as:

$$[C^e_{t+1}]:\ \xi_{\chi_e}\chi_e\left(C^e_{t+1}\right)^{(\xi_{\chi_e}\chi_e-1)}\left(N^e_{t+1}\right)^{1-\xi_{\chi_e}\chi_e}-\lambda^{\chi_e}_t=0$$

$$[N^e_{t+1}]:\ (1-\xi_{\chi_e}\chi_e)\left(C^e_{t+1}\right)^{\xi_{\chi_e}\chi_e}\left(N^e_{t+1}\right)^{-\xi_{\chi_e}\chi_e}-\lambda^{\chi_e}_t=0$$

$$[\lambda^{\chi_e}_t]:\ C^e_{t+1}+N^e_{t+1}-\Psi^e_{t+1}=0$$

From first order conditions we get the following optimal rules

$$C^e_{t+1}=\chi_e\Psi^e_{t+1}$$
$$N^e_{t+1}=(1-\chi_e)\,\Psi^e_{t+1}$$

In their first period, entrepreneurs will try to maximize expected second period wealth, Ψ^e_{t+1}, by purchasing capital at nominal price Q^K_t, which will be productive (and rented) in the next period. These purchases are financed using the resources left as bequests by the previous generation of entrepreneurs and borrowing an amount L^F_t at nominal rate R^L_t from from F banks. In borrowing from banks, entrepreneurs also face an agency problem of the type faced by impatient households i.e. in $t+1$ entrepreneurs receive an idiosyncratic shock to the efficiency units of capital that will ultimately determine their ability to pay their liabilities to banks. Banks cannot observe these shock, but entrepreneurs can. Depreciated capital is sold in the next period to capital producers at Q^K_{t+1}. Entrepreneurial leverage, as measured by assets over equity, is $lev^e_t = {}^{Q^K_t K_t}/_{N^e_t}$.

In this setting, entrepreneurs solve, in their first period,

$$\max_{K_t,L^F_t} \mathbb{E}_t\left(\Psi^e_{t+1}\right) \text{ subject to}$$

$$Q^K_t K_t - L^F_t = N^e_t$$
$$\Psi^e_{t+1}=\max\left[\omega^e_{t+1}\left(R^k_{t+1}+(1-\delta_K)\,Q^K_{t+1}\right)K_t - R^L_t L^F_t,\ 0\right]$$

and a bank participation condition, which will be explained later. The factor ω^e_{t+1} represents the idiosyncratic shock to the entrepreneurs efficiency units of capital. This shock takes place after the loan with the bank has taken place but before renting capital to consumption goods producers. It is assumed that this shock is independently and identically distributed across entrepreneurs and follows a log-normal distribution with an expected value of one. Let

$$R^e_{t+1}=\left[\frac{R^k_{t+1}+(1-\delta_K)\,Q^K_{t+1}}{Q^K_t}\right] \tag{62}$$

be the gross nominal return per efficiency unit of capital obtained in period $t+1$ from capital obtained in period t. Then in order for the entrepreneur to pay for its loan the efficiency shock, ω^e_{t+1}, must exceed the threshold

$$\bar{\omega}^e_{t+1}=\frac{R^L_t L^F_t}{R^e_{t+1}Q^K_t K_t}$$

If $\omega^e_{t+1}\geq\bar{\omega}^e_{t+1}$ the entrepreneurs pays $R^L_t L^F_t$ to the bank and gets $(\omega^e_{t+1}-\bar{\omega}^e_{t+1})R^e_{t+1}Q^K_t K_t$. Otherwise, the entrepreneurs defaults and receives nothing. While F-banks only recover $(1-\mu_e)\omega^e_{t+1}R^e_{t+1}Q^K_t K_t$ from non performing loans, and $R^L_t L^F_t$ from performing loans. With the threshold, we can define $PD^e_t = F_e(\bar{\omega}^e_t)$ as the default rate of entrepreneurs on their loans.

The share of the gross return that goes to the bank is denoted as $\Gamma_e(\bar{\omega}^e_{t+1})$ whereas the share of gross return that goes to the entrepreneur is $(1-\Gamma_e(\bar{\omega}^e_{t+1}))$ where:

$$\Gamma_e\left(\bar{\omega}^e_{t+1}\right)=\int_0^{\bar{\omega}^e_{t+1}}\omega^e_{t+1}f_e\left(\omega^e_{t+1}\right)d\omega^e_{t+1}+\bar{\omega}^e_{t+1}\int_{\bar{\omega}^e_{t+1}}^{\infty}f_e\left(\omega^e_{t+1}\right)d\omega^e_{t+1}$$

also let

$$G_e\left(\bar{\omega}^e_{t+1}\right)=\int_0^{\bar{\omega}^e_{t+1}}\omega^e_{t+1}f_e\left(\omega^e_{t+1}\right)d\omega^e_{t+1}$$

denote the part of those returns that come from the defaulted loans. Taking into consideration the share of the return that is lost due to verification cost as $\mu_e G_e\left(\bar{\omega}^e_{t+1}\right)$, then the net share of return that goes to the bank is

$$\Gamma_e\left(\bar{\omega}^e_{t+1}\right) - \mu_e G_e\left(\bar{\omega}^e_{t+1}\right).$$

Taking this into account then the maximization problem of the entrepreneur can be written as

$$\max_{\bar{\omega}^e_{t+1}, K_t} \mathbb{E}_t\left\{\Psi^e_{t+1}\right\} = \mathbb{E}_t\left\{\left[1-\Gamma_e\left(\bar{\omega}^e_{t+1}\right)\right] R^e_{t+1} Q^K_t K_t\right\}, \text{ subject to}$$

$$\mathbb{E}_t\left\{\left[1-\Gamma_F\left(\bar{\omega}^F_{t+1}\right)\right]\left[\Gamma_e\left(\bar{\omega}^e_{t+1}\right) - \mu_e G_e\left(\bar{\omega}^e_{t+1}\right)\right] R^e_{t+1} Q^K_t K_t\right\} \geq \rho^F_{t+1}\phi_F L^F_t, \tag{63}$$

that says that banks will participate in the contract only if its net expected profits are at least equal to their alternative use of funds. This yields the following optimality conditions

$$\left(1-\Gamma^e_{t+1}\right) = \lambda^e_t\left(\frac{\rho^F_{t+1}\phi^F_t}{R^e_{t+1}} - \left(1-\Gamma^F_{t+1}\right)\left[\Gamma^e_{t+1} - \mu^e G^e_{t+1}\right]\right) \tag{64}$$

$$\Gamma^{e\prime}_{t+1} = \lambda^e_t\left(1-\Gamma^F_{t+1}\right)\left[\Gamma^{e\prime}_{t+1} - \mu^e G^{e\prime}_{t+1}\right] \tag{65}$$

Further, it is assumed that $\ln(\omega^e_t) \sim N(-0.5(\sigma^e_t)^2, (\sigma^e_t)^2)$, leading to analogous properties as with impatient households for $\bar{\omega}^e_t$, Γ_e and G_e.

A.3 Bankers and Banks

A.3.1 Bankers

Bankers are modeled as in Clerc et al. (2014) and in a similar way to entrepreneurs: They belong to a sequence of overlapping generations of risk-neutral agents who live 2 periods and have exclusive access to the opportunity of investing their wealth as banks' inside equity capital.

In the first period, the banker receives a bequest N^b_t from the previous generation of bankers and must distribute it across the two types of existing banks: banks specializing in corporate loans (F banks) and banks specializing in housing loans (H banks). That is, a banker who chooses to invest an amount E^F_t of inside equity in F banks will invest the rest of her bequest in H banks, $E^H_t = N^b_t - E^F_t$. Then, in the second period bankers receive their returns from both investments, and must choose how to distribute their net worth Ψ^b_{t+1} between transferring dividends C^b_{t+1} to households and leaving bequests N^b_{t+1} to the next generation. Additionally, disturbances to the exogenous variable $\xi^{\chi_b}_t$ capture transitory fluctuations in the banker's dividend policy

Given Ψ^b_{t+1}, the banker will distribute it by solving the following maximization problem:

$$\max_{C^b_{t+1}, N^b_{t+1}} \left(C^b_{t+1}\right)^{\xi^{\chi_b}_{t+1}\chi^b}\left(N^b_{t+1}\right)^{1-\xi^{\chi_b}_{t+1}\chi^b}, \text{ subject to}$$

$$C^b_{t+1} + N^b_{t+1} = \Psi^b_{t+1}$$

which leads to the following optimal rules

$$C^b_{t+1} = \xi^{\chi_b}_{t+1}\chi^b\Psi^b_{t+1} \tag{66}$$

$$N^b_{t+1} = \left(1-\xi^{\chi_b}_{t+1}\chi^b\right)\Psi^b_{t+1} \tag{67}$$

In turn, net worth in the second period is determined by the returns on bankers' investments in period-t:

$$\Psi^b_{t+1} = \rho^F_{t+1}E^F_t + \xi^{b,roe}_t\rho^H_{t+1}\left(N^b_t - E^F_t\right)$$

where $\xi^{b,roe}_t$ is a shock to the bankers' required return to equity invested in the housing branches, ρ^j_{t+1} is the period $t+1$ ex-post gross return on inside equity E^j_t invested in period t in bank of class j. In order to capture the fact that most of mortgage debt takes the form of non endorsable debt —meaning the issuer bank retains it in its balance sheet to maturity— we assume that the banker $j = H$ invests in the banking project H through a mutual fund which pays the expected average return to housing equity ρ^H_{t+1} every period. Thus, letting $\tilde{\rho}^H_t$ represent the period return on housing portfolio, then $\rho^H_t = \kappa\tilde{\rho}^H_t + (1-\kappa)\rho^H_{t+1}$. The banker then chooses

$$\max_{E^F_t} \mathbb{E}_t\left\{\Psi^b_{t+1}\right\} = \mathbb{E}_t\left\{\rho^F_{t+1}E^F_t + \xi^{b,roe}_t\rho^H_{t+1}\left(N^b_t - E^F_t\right)\right\}$$

An interior equilibrium in which both classes of banks receive strictly positive inside equity from bankers will require the following equality to hold:

$$\mathbb{E}_t\left\{\rho^F_{t+1}\right\} = \mathbb{E}_t\left\{\xi^{b,roe}_t \rho^H_{t+1}\right\} = \bar{\rho}_t$$

where $\bar{\rho}_t$ denotes banks' required expected gross rate of return on equity investment undertaken at time t.

A.3.2 Banks

Banks are institutions specialized in extending either corporate or housing loans drawing funds through deposits, and bonds from unconstrained household, and equity from bankers. We assume a continuum of identical banking institutions of j class banks $j = \{F, H\}$. In particular, banks of class j are investment projects created in period-t that in $t+1$ generate profits Π^j_{t+1} before being liquidated with:

$$\Pi^F_{t+1} = \max\left[\omega^F_{t+1}\tilde{R}^F_{t+1}L^F_t - R^D_t D^F_t, 0\right], \quad \Pi^H_{t+1} = \max\left[\omega^H_{t+1}\tilde{R}^H_{t+1}Q^L_t L^H_t - R^{BB}_{t+1}Q^{BB}_{t+1}BB_t, 0\right]$$

where $\tilde{R}^j_{t+1}$ is the realized return on a well-diversified portfolio of loans to entrepreneurs or households and ω^j_{t+1} is an idiosyncratic portfolio return shock, which is i.i.d across banks of class j with a cdf of $F_j(\omega^j_{t+1})$ and pdf $f_j(\omega^j_{t+1})$. Due to limited liability, the equity payoff may not be negative, which defines thresholds $\bar{\omega}^j_{t+1}$:

$$\bar{\omega}^F_{t+1} \equiv \frac{R^D_t D^F_t}{\tilde{R}^F_{t+1}L^F_t}, \qquad \bar{\omega}^H_{t+1} \equiv \frac{R^{BB}_{t+1}Q^{BB}_{t+1}BB_t}{\tilde{R}^H_{t+1}Q^L_t L^H_t}$$

Similar to households and entrepreneurs, $\Gamma_j(\bar{\omega}^j_{t+1})$ denotes the share of gross returns to bank j investments which are either paid back to depositors or bond holders, implying that $[1 - \Gamma_j(\bar{\omega}^j_{t+1})]$ is the share that the banks will keep as profits. We also define $G_j(\bar{\omega}^j_{t+1})$ as the share of bank j assets which belong to defaulting j banks, and thus $\mu_j G_j(\bar{\omega}^j_{t+1})$ is the total cost of bank j defaults expressed as a fraction of total bank j assets.

The balance sheet of banks of class F is given by $L^F_t = E^F_t + D^F_t$, and they face a regulatory capital constraint given by $E^F_t \geq \phi_F L^F_t$, where ϕ_F is the capital-to-asset ratio, and is binding at all times in equilibrium so that the loans can be written as $L^F_t = {E^F_t}/{\phi_F}$ and the deposits as $D^F_t = ({1-\phi_F}/{\phi_F})E^F_t$. Likewise, balance sheet of banks of class H is given by $Q^L_t L^H_t = E^H_t + Q^{BB}_t BB_t$, with binding capital regulation determining $E^H_t = \phi_H Q^L_t L^H_t$, and $Q^{BB}_t BB_t = {(1-\phi_H)}/{\phi_H}E^H_t$. Further, using the threshold definitions and the binding capital constraints, we obtain:

$$\bar{\omega}^F_{t+1} = (1-\phi_F)\frac{R^D_t}{\tilde{R}^F_{t+1}}$$

$$\bar{\omega}^H_{t+1} = (1-\phi_H)\frac{R^{BB}_{t+1}}{\tilde{R}^H_{t+1}}\left(\frac{Q^{BB}_{t+1}}{Q^{BB}_t}\right)$$

Finally, we define the realized rate of return of equity invested in a bank of class j:

$$\rho^j_{t+1} = \left[1 - \Gamma_j\left(\bar{\omega}^j_{t+1}\right)\right]\frac{\tilde{R}^j_{t+1}}{\phi_j} \tag{68}$$

For completeness, notice that derivations in prior sections imply that following expressions for $\tilde{R}^j_{t+1}$, $j = \{F, H\}$:

$$\tilde{R}^F_{t+1} = \left(\Gamma_e\left(\bar{\omega}^e_{t+1}\right) - \mu_e G_e\left(\bar{\omega}^e_{t+1}\right)\right)\frac{R^e_{t+1}Q^K_t K_t}{L^F_t}$$

$$\tilde{R}^H_{t+1} = \left(\Gamma_I\left(\bar{\omega}^I_{t+1}\right) - \mu_I G_I\left(\bar{\omega}^I_{t+1}\right)\right)\frac{R^H_{t+1}Q^H_t H^I_t}{Q^L_t L^H_t}$$

As with households and entrepreneurs, it is assumed that the bank idiosyncratic shock follows a log-normal distribution: $\log(\omega^j_t) \sim N(-\frac{1}{2}(\sigma^j_t)^2, (\sigma^j_t)^2)$, leading to analogous properties for $\bar{\omega}^j_t$, Γ_j and G_j.

A.4 Production

The supply side of the economy is composed by different types of firms that are all owned by the households. Monopolistically competitive unions act as wage setters by selling household's differentiated varieties of labor supply n_{it} to a perfectly competitive firm, which packs these varieties into a composite labor service $\widetilde{n}_t$. There is a set of monopolistically competitive firms producing different varieties of a home good, Y^H_{jt}, using wholesale good X^Z_t as input; a set of monopolistically competitive importing firms that import a homogeneous foreign good to transform it into varieties, X^F_{jt}; and three groups of perfectly competitive firms that aggregate products: one packing different varieties of the home good into a composite home good, X^H_t, one packing the imported varieties into a composite foreign good, X^F_t, and, finally, another one that bundles the composite home and foreign goods to create a final good, Y^C_t. This final good is purchased by households (C^P_t,C^I_t), capital and housing producers (I^K_t,I^H_t), and the government (G_t).

Similarly to Clerc et al. (2014), we model perfectly competitive capital-producing and housing-producing firms. Both types of firms are owned by patient households and their technology is subject to an adjustment cost. They produce new units of capital and housing from the final good and sell them to entrepreneurs and households respectively. However, we depart from Clerc et al. (2014) by assuming time-to-build frictions in housing investment. Finally, there is a set of competitive firms producing a homogeneous commodity good that is exported abroad (and which follows an exogenous process). The total mass of firms in each sector is normalized to one.

A.4.1 Capital goods

There is a continuum of competitive capital firm producers who buy an amount I_t of final goods at price P_t and use their technology to satisfy the demand for new capital goods not covered by depreciated capital, i.e. $K_t - (1-\delta_K)K_{t-1}$, where new units of capital are sold at price Q^K_t. As is usual in the literature, we assume that the aggregate stock of new capital considers investment adjustment costs and evolves according to following law of motion:

$$K_t = (1-\delta_K)\,K_{t-1} + \left[1 - \frac{\gamma_K}{2}\left(\frac{I_t}{I_{t-1}} - a\right)^2\right]\xi^i_t I_t$$

Where ξ^i_t is a shock to investment efficiency. Therefore a representative capital producer chooses how much to invest in order to maximize the discounted utility of its profits,

$$\sum_{i=0}^{\infty} r_{t,t+i}\left\{Q^K_{t+i}\left[1 - \frac{\gamma_K}{2}\left(\frac{I_{t+i}}{I_{t+i-1}} - a\right)^2\right]\xi^i_{t+i}I_{t+i} - P_{t+i}I_{t+i}\right\}$$

Discounting is done according to patient households' preferences, who are the owners of the firms. From the first order condition a new relation can be obtained that relates the price of capital to the level of investment

$$\begin{aligned} P_t \;&=\; Q^K_t\left\{\left(1 - \frac{\gamma_K}{2}\left(\frac{I_t}{I_{t-1}} - a\right)^2\right) - \gamma_K\left(\frac{I_t}{I_{t-1}} - a\right)\frac{I_t}{I_{t-1}}\right\}\xi^i_t \\ &\quad + E_t\left\{r_{t,t+1}Q^K_{t+1}\gamma_K\left(\frac{I_{t+1}}{I_t} - a\right)\left(\frac{I_{t+1}}{I_t}\right)^2\xi^i_{t+1}\right\} \end{aligned} \tag{69}$$

A.4.2 Housing goods

The structure of housing producers is similar to that of capital good producers with the difference that housing goods also face investment adjustment costs in the form of time to build Kydland and Prescott (1982) and Uribe and Yue (2006). As such, there is a continuum of competitive housing firm producers who authorize housing investment projects I^{AH}_t in period t, which will increase housing stock N_H periods later, the time it takes to build.[9] Thus, the law of motion for the aggregate stock of housing in H_t will consider projects authorized N_H periods before, and includes investment adjustment costs,

$$H_t = (1-\delta_H)\,H_{t-1} + \left[1 - \frac{\gamma_H}{2}\left(\frac{I^{AH}_{t-N_H}}{I^{AH}_{t-N_H-1}} - a\right)^2\right]\xi^{ih}_{t-N_H}I^{AH}_{t-N_H}$$

where ξ^{ih}_t is a shock to housing investment efficiency, and the sector covers all demand for new housing, $H_t - (1-\delta_H)H_{t-1}$, by selling units at price Q^H_t.

The firm's effective expenditure is spread out during the periods that new housing is being built. In particular, the amount of final goods purchased (at price P_t) by the firm in t to produce housing is given by

$$I^H_t = \sum_{j=0}^{N_H}\varphi^H_j I^{AH}_{t-j}$$

Where φ^H_j (the fraction of projects authorized in period $t-j$ that is outlaid in period t) satisfy $\sum_{j=0}^{N_H}\varphi^H_j = 1$ and $\varphi^H_j =$

[9] Notice that if $N_H = 0$, the structure is symmetric to the capital producers.

$\rho^{\varphi H}\varphi^H_{j-1}$.[10]

Therefore a representative housing producer chooses how much to authorize in new projects I^{AH}_t in order to maximize the discounted utility of its profits,

$$\sum_{i=0}^{\infty} r_{t,t+i}\left\{Q^H_{t+i}\left[1-\frac{\gamma_H}{2}\left(\frac{I^{AH}_{t-N_H+i}}{I^{AH}_{t-N_H+i-1}}-a\right)^2\right]\xi^{ih}_{t-N_H+i}I^{AH}_{t-N_H+i}-P_{t+i}I^H_{t+i}\right\}$$

Where discounting is done according to patient households' preferences, who are the owners of the firms. From the first order condition a new relation can be obtained that relates the price of housing to the level of housing investment

$$\begin{aligned} E_t\sum_{j=0}^{N_H} r_{t,t+j}\varphi^H_j P_{t+j} &= E_t r_{t,t+N_H}Q^H_{t+N_H}\left\{\left[1-\frac{\gamma_H}{2}\left(\frac{I^{AH}_t}{I^{AH}_{t-1}}-a\right)^2\right]-\gamma_H\left(\frac{I^{AH}_t}{I^{AH}_{t-1}}-a\right)\frac{I^{AH}_t}{I^{AH}_{t-1}}\right\}\xi^{ih}_t \\ &\quad +E_t r_{t,t+N_H+1}Q^H_{t+N_H+1}\left\{\gamma_H\left(\frac{I^{AH}_{t+1}}{I^{AH}_t}-a\right)\left(\frac{I^{AH}_{t+1}}{I^{AH}_t}\right)^2\xi^{ih}_{t+1}\right\} \end{aligned} \tag{70}$$

A.4.3 Final goods

A representative final goods firm demands composite home and foreign goods in the amounts X^H_t and X^F_t, respectively, and combines them according to the following technology:

$$Y^C_t=\left[\omega^{1/\eta}\left(X^H_t\right)^{1-1/\eta}+(1-\omega)^{1/\eta}\left(X^F_t\right)^{1-1/\eta}\right]^{\frac{\eta}{\eta-1}} \tag{71}$$

where $\omega\in(0,1)$ is inversely related to the degree of home bias and $\eta>0$ measures the substitutability between domestic and foreign goods. The selling price of this final good is denoted by P_t, while the prices of the domestic and foreign inputs are P^H_t and P^F_t, respectively. Subject to the technology constraint (71), the firm maximizes its profits over the inputs, taking prices as given:

$$\max_{X^H_t,X^F_t}\; P_t\left[\omega^{1/\eta}\left(X^H_t\right)^{1-1/\eta}+(1-\omega)^{1/\eta}\left(X^F_t\right)^{1-1/\eta}\right]^{\frac{\eta}{\eta-1}}-P^H_tX^H_t-P^F_tX^F_t$$

The first-order conditions of this problem determine the optimal input demands:

$$X^H_t=\omega\left(\frac{P^H_t}{P_t}\right)^{-\eta}Y^C_t \tag{72}$$

$$X^F_t=(1-\omega)\left(\frac{P^F_t}{P_t}\right)^{-\eta}Y^C_t \tag{73}$$

Combining these optimality conditions and using that zero profits hold in equilibrium, we can write

$$P_t=\left[\omega\left(P^H_t\right)^{1-\eta}+(1-\omega)\left(P^F_t\right)^{1-\eta}\right]^{\frac{1}{1-\eta}} \tag{74}$$

A.4.4 Home composite goods

A representative home composite goods firm demands home goods of all varieties $j\in[0,1]$ in amounts X^H_{jt} and combines them according to the technology

$$Y^H_t=\left[\int_0^1\left(X^H_{jt}\right)^{\frac{\epsilon_H-1}{\epsilon_H}}dj\right]^{\frac{\epsilon_H}{\epsilon_H-1}} \tag{75}$$

with $\epsilon_H>0$. Let P^H_{jt} denote the price of the home good of variety j. Subject to the technology constraint (75), the firm maximizes its profits $\Pi^H_t=P^H_tY^H_t-\int_0^1P^H_{jt}X^H_{jt}dj$ over the input demands X^H_{jt} taking prices as given:

$$\max_{X^H_{jt}}\; P^H_t\left[\int_0^1\left(X^H_{jt}\right)^{\frac{\epsilon_H-1}{\epsilon_H}}dj\right]^{\frac{\epsilon_H}{\epsilon_H-1}}-\int_0^1P^H_{jt}X^H_{jt}dj$$

[10] Notice that $\rho^{\varphi H}>1$ implies that expenditure for any authorized project is back-loaded (increasing over time), while the converse is true for $\rho^{\varphi H}<1$.

This implies the following first-order conditions for all j:

$$\partial X_{jt}^H : \ P_t^H \left(Y_t^H\right)^{1/\epsilon_H} \left(X_{jt}^H\right)^{-1/\epsilon_H} - P_{jt}^H = 0$$

such that the input demand functions are

$$X_{jt}^H = \left(\frac{P_{jt}^H}{P_t^H}\right)^{-\epsilon_H} Y_t^H \tag{76}$$

Substituting (76) into (75) yields the price of home composite goods:

$$P_t^H = \left[\int_0^1 \left(P_{jt}^H\right)^{1-\epsilon_H} dj\right]^{\frac{1}{1-\epsilon_H}} \tag{77}$$

A.4.5 Home goods of variety j

There is a continuum of j's firms, with measure one, that demand a domestic wholesale good X_t^Z and differentiate into home goods varieties Y_{jt}^H. To produce one unit of variety j, firms need one unit of input according to

$$\int_0^1 Y_{jt}^H dj = X_t^Z \tag{78}$$

The firm producing variety j satisfies the demand given by (76) but it has monopoly power for its variety. For varieties, the nominal marginal cost in terms of the composite good price is given by $P_t^H mc_{jt}^H$. Given that, every firm buys their input from the same wholesale market. It implies that all of them face the same nominal marginal costs

$$P_t^H mc_{jt}^H = P_t^H mc_t^H = P_t^Z \tag{79}$$

Given nominal marginal costs $P_t^H mc_{jt}^H$, firm j chooses its price P_{jt}^H to maximize profits. In setting prices, the firm faces a Calvo-type problem, whereby each period the firm can change its price optimally with probability $1-\theta_H$, and if it cannot optimally change its price, it indexes its previous price according to a weighted product of past and steady state inflation with weights $\kappa_H \in [0,1]$ and $1-\kappa_H$ respectively. A firm reoptimizing in period t will choose the price $\tilde{P}_{jt}^H$ that maximizes the current market value of the profits generated until it can reoptimize again. [11] As the firms are owned by the households, profits are discounted using the households' stochastic discount factor for nominal payoffs, $r_{t,t+s}$. A reoptimizing firm, therefore, solves the following problem:

$$\max_{\tilde{P}_{jt}^H} \ E_t \sum_{s=0}^{\infty} \theta_H^s r_{t,t+s} \left(P_{jt+s}^H - P_{t+s}^H mc_{jt+s}^H\right) Y_{jt+s}^H \quad \text{s.t.} \quad Y_{jt+s}^H = X_{jt+s}^H = \left(\frac{\tilde{P}_{jt}^H \Pi_{i=1}^s \pi_{t+i}^{I,H}}{P_{t+s}^H}\right)^{-\epsilon_H} Y_{t+s}^H$$

which can be rewritten as

$$\max_{\tilde{P}_{jt}^H} \ E_t \sum_{s=0}^{\infty} \theta_H^s r_{t,t+s} \left[\left(\tilde{P}_{jt}^H \Pi_{i=1}^s \pi_{t+i}^{I,H}\right)^{1-\epsilon_H} \left(P_{t+s}^H\right)^{\epsilon_H} - mc_{jt+s}^H \left(\tilde{P}_{jt}^H \Pi_{i=1}^s \pi_{t+i}^{I,H}\right)^{-\epsilon_H} \left(P_{t+s}^H\right)^{1+\epsilon_H}\right] Y_{t+s}^H$$

[11] Therefore, the following relation holds: $P_{jt+s}^H = \tilde{P}_{jt}^H \pi_{t+1}^{I,H} \ldots \pi_{t+s}^{I,H}$, where $\pi_t^{I,H} = \left(\pi_{t-1}^H\right)^{\kappa_H} \left(\pi_t^T\right)^{1-\kappa_H}$, $\pi_t^H = P_t^H / P_{t-1}^H$, and π_t^T denotes the inflation target in period t.

The first-order conditions determining the optimal price $\tilde{P}_t^H$ can be written as follows:[12]

$$\begin{aligned}
0 &= E_t \sum_{s=0}^{\infty} \theta_H^s r_{t,t+s} \Big[(1-\epsilon_H) \left(\tilde{P}_t^H\right)^{-\epsilon_H} \left(\Pi_{i=1}^s \pi_{t+i}^{I,H}\right)^{1-\epsilon_H} \left(P_{t+s}^H\right)^{\epsilon_H} \\
&\qquad + \epsilon_H mc_{t+s}^H \left(\tilde{P}_t^H\right)^{-\epsilon_H-1} \left(\Pi_{i=1}^s \pi_{t+i}^{I,H}\right)^{-\epsilon_H} \left(P_{t+s}^H\right)^{1+\epsilon_H} \Big] Y_{t+s}^H \\
\Leftrightarrow \quad 0 &= E_t \sum_{s=0}^{\infty} \theta_H^s r_{t,t+s} \Bigg[\frac{\epsilon_H - 1}{\epsilon_H} \left(\tilde{P}_t^H \Pi_{i=1}^s \pi_{t+i}^{I,H}\right)^{1-\epsilon_H} \frac{\left(P_{t+s}^H\right)^{\epsilon_H}}{P_t^H} \\
&\qquad - mc_{t+s}^H \left(\tilde{P}_t^H \Pi_{i=1}^s \pi_{t+i}^{I,H}\right)^{-\epsilon_H} \frac{\left(P_{t+s}^H\right)^{1+\epsilon_H}}{P_t^H} \Bigg] Y_{t+s}^H \\
\Leftrightarrow \quad 0 &= E_t \sum_{s=0}^{\infty} \theta_H^s r_{t,t+s} \Bigg[\frac{\epsilon_H - 1}{\epsilon_H} \left(\tilde{p}_t^H \Pi_{i=1}^s \pi_{t+i}^{I,H}\right)^{1-\epsilon_H} \left(\frac{P_{t+s}^H}{P_t^H}\right)^{\epsilon_H} \\
&\qquad - mc_{t+s}^H \left(\tilde{p}_t^H \Pi_{i=1}^s \pi_{t+i}^{I,H}\right)^{-\epsilon_H} \left(\frac{P_{t+s}^H}{P_t^H}\right)^{1+\epsilon_H} \Bigg] Y_{t+s}^H
\end{aligned}$$

where the second step follows from multiplying both sides by $-\tilde{P}_t^H/(P_t^H \epsilon_H)$, and the third by defining $\tilde{p}_t^H = \tilde{P}_t^H / P_t^H$. The first-order condition can be rewritten in recursive form as follows, defining $F_t^{H_1}$ as

$$\begin{aligned}
F_t^{H_1} &= \frac{\epsilon_H - 1}{\epsilon_H} \left(\tilde{p}_t^H\right)^{1-\epsilon_H} Y_t^H + E_t \sum_{s=1}^{\infty} \theta_H^s r_{t,t+s} \frac{\epsilon_H - 1}{\epsilon_H} \left(\tilde{p}_t^H \Pi_{i=1}^s \pi_{t+i}^{I,H}\right)^{1-\epsilon_H} \left(\frac{P_{t+s}^H}{P_t^H}\right)^{\epsilon_H} Y_{t+s}^H \\
&= \frac{\epsilon_H - 1}{\epsilon_H} \left(\tilde{p}_t^H\right)^{1-\epsilon_H} Y_t^H + E_t \sum_{s=0}^{\infty} \theta_H^{s+1} r_{t,t+s+1} \frac{\epsilon_H - 1}{\epsilon_H} \left(\tilde{p}_t^H \Pi_{i=1}^{s+1} \pi_{t+i}^{I,H}\right)^{1-\epsilon_H} \left(\frac{P_{t+s+1}^H}{P_t^H}\right)^{\epsilon_H} Y_{t+s+1}^H \\
&= \frac{\epsilon_H - 1}{\epsilon_H} \left(\tilde{p}_t^H\right)^{1-\epsilon_H} Y_t^H + \theta_H E_t \Bigg\{ r_{t,t+1} \left(\frac{\tilde{p}_t^H \pi_{t+1}^{I,H}}{\tilde{p}_{t+1}^H}\right)^{1-\epsilon_H} \left(\pi_{t+1}^H\right)^{\epsilon_H} \sum_{s=0}^{\infty} \theta_H^s r_{t+1,t+s+1} \frac{\epsilon_H - 1}{\epsilon_H} \\
&\qquad \times \left(\tilde{p}_{t+1}^H \Pi_{i=1}^s \pi_{t+1+i}^{I,H}\right)^{1-\epsilon_H} \left(\frac{P_{t+s+1}^H}{P_{t+1}^H}\right)^{\epsilon_H} Y_{t+s+1}^H \Bigg\} \\
&= \frac{\epsilon_H - 1}{\epsilon_H} \left(\tilde{p}_t^H\right)^{1-\epsilon_H} Y_t^H + \theta_H E_t \Bigg\{ r_{t,t+1} \left(\frac{\tilde{p}_t^H \pi_{t+1}^{I,H}}{\tilde{p}_{t+1}^H}\right)^{1-\epsilon_H} \left(\pi_{t+1}^H\right)^{\epsilon_H} F_{t+1}^{H_1} \Bigg\}
\end{aligned} \tag{80}$$

and, analogously, $F_t^{H_2}$ as

$$\begin{aligned}
F_t^{H_2} &= \left(\tilde{p}_t^H\right)^{-\epsilon_H} mc_t^H Y_t^H + E_t \sum_{s=1}^{\infty} \theta_H^s r_{t,t+s} mc_{t+s}^H \left(\tilde{p}_t^H \Pi_{i=1}^s \pi_{t+i}^{I,H}\right)^{-\epsilon_H} \left(\frac{P_{t+s}^H}{P_t^H}\right)^{1+\epsilon_H} Y_{t+s}^H \\
&= \left(\tilde{p}_t^H\right)^{-\epsilon_H} mc_t^H Y_t^H + E_t \sum_{s=0}^{\infty} \theta_H^{s+1} r_{t,t+s+1} mc_{t+s+1}^H \left(\tilde{p}_t^H \Pi_{i=1}^{s+1} \pi_{t+i}^{I,H}\right)^{-\epsilon_H} \left(\frac{P_{t+s+1}^H}{P_t^H}\right)^{1+\epsilon_H} Y_{t+s+1}^H \\
&= \left(\tilde{p}_t^H\right)^{-\epsilon_H} mc_t^H Y_t^H + \theta_H E_t \Bigg\{ r_{t,t+1} \left(\frac{\tilde{p}_t^H \pi_{t+1}^{I,H}}{\tilde{p}_{t+1}^H}\right)^{-\epsilon_H} \left(\pi_{t+1}^H\right)^{1+\epsilon_H} \sum_{s=0}^{\infty} \theta_H^s r_{t+1,t+s+1} mc_{t+s+1}^H \\
&\qquad \times \left(\tilde{p}_{t+1}^H \Pi_{i=1}^s \pi_{t+1+i}^{I,H}\right)^{-\epsilon_H} \left(\frac{P_{t+s+1}^H}{P_{t+1}^H}\right)^{1+\epsilon_H} Y_{t+s+1}^H \Bigg\} \\
&= \left(\tilde{p}_t^H\right)^{-\epsilon_H} mc_t^H Y_t^H + \theta_H E_t \Bigg\{ r_{t,t+1} \left(\frac{\tilde{p}_t^H \pi_{t+1}^{I,H}}{\tilde{p}_{t+1}^H}\right)^{-\epsilon_H} \left(\pi_{t+1}^H\right)^{1+\epsilon_H} F_{t+1}^{H_2} \Bigg\}
\end{aligned} \tag{81}$$

such that

$$F_t^{H_1} = F_t^{H_2} = F_t^H \tag{82}$$

[12] Notice that the subscript j has been removed from $\tilde{P}_t^H$; this simplifies notation and underlines that the prices chosen by all firms j that reset prices optimally in a given period are equal as they face the same problem by (79).

Using (77), we have

$$
\begin{aligned}
1 &= \int_0^1 \left(\frac{P^H_{jt}}{P^H_t}\right)^{1-\epsilon_H} dj \\
&= (1-\theta_H)\left(\tilde{p}^H_t\right)^{1-\epsilon_H} + \theta_H \left(\frac{P^H_{t-1}\pi^{I,H}_t}{P^H_t}\right)^{1-\epsilon_H} \\
&= (1-\theta_H)\left(\tilde{p}^H_t\right)^{1-\epsilon_H} + \theta_H \left(\frac{\pi^{I,H}_t}{\pi^H_t}\right)^{1-\epsilon_H} \qquad (83)
\end{aligned}
$$

The second equality above follows from the fact that, under Calvo pricing, the distribution of prices among firms not reoptimizing in period t corresponds to the distribution of aggregate prices in period $t-1$, though with total mass reduced to θ_H.

A.4.6 Wholesale Domestic Goods

There is a representative firm producing a homogeneous wholesale home good, combining capital and labor according to the following technology:

$$
Y^Z_t = z_t K^{\alpha}_{t-1}\left(A_t \widetilde{n}_t\right)^{1-\alpha} \qquad (84)
$$

with capital share $\alpha \in (0,1)$, an exogenous stationary technology shock z_t and a non-stationary technology A_t. Production of the wholesale good composite labor services $\widetilde{n}_t$ and capital K_{t-1}. Additionally, following Lechthaler et al. (2010), the firm faces a quadratic adjustment costs of labor which is a function of parameter γ_n, and of aggregate wholesale domestic goods $\widetilde{Y}^Z_t$, which in equilibrium are equal to Y^Z_t and which the representative firm takes as given. In a first stage, the firm hires composite labor and rents capital to solve the following problem:

$$
\min_{\widetilde{n}_{t+s},K_{t+s-1}} \sum_{s=0}^{\infty} r_{t,t+s}\left\{W_{t+s}\widetilde{n}_{t+s} + \frac{\gamma_n}{2}\left(\frac{\widetilde{n}_{t+s}}{\widetilde{n}_{t+s-1}}-1\right)^2 \widetilde{Y_{t+s}}^Z P^Z_t + R_t K_{t+s-1}\right\}
$$
$$
s.t. \quad Y^Z_{t+s} = X^Z_{t+s} = z_{t+s}K^{\alpha}_{t+s-1}\left(A_{t+s}\widetilde{n}_{t+s}\right)^{1-\alpha}
$$

Then, the optimal capital and labor demands are given by:

$$
\widetilde{n}_t = (1-\alpha)\left\{\frac{mc^Z_t Y^Z_t}{W_t + \gamma_n\left(\frac{\widetilde{n}_t}{\widetilde{n}_{t-1}}-1\right)\left(\frac{1}{\widetilde{n}_{t-1}}\right)\widetilde{Y}^Z_t P^Z_t - r_{t,t+1}\gamma_n \mathbb{E}_t\left(\frac{\widetilde{n}_{t+1}}{\widetilde{n}_t}-1\right)\left(\frac{\widetilde{n}_{t+1}}{\widetilde{n}^2_t}\right)\widetilde{Y}^Z_{t+1}P^Z_{t+1}}\right\} \qquad (85)
$$

$$
K_{t-1} = \alpha\left(\frac{mc^Z_t}{R^k_t}\right)Y^Z_t \qquad (86)
$$

Where mc^Z_t is the lagrangian multiplier on the production function and $r_{t,t+1}$ the households' stochastic discount factor between periods t and $t+1$. The, combining both optimality conditions:

$$
\frac{K_{t-1}}{\widetilde{n}_t} = \frac{\alpha}{(1-\alpha)R^k_t}\left\{W_t + \gamma_n\left(\frac{\widetilde{n}_t}{\widetilde{n}_{t-1}}-1\right)\left(\frac{1}{\widetilde{n}_{t-1}}\right)\widetilde{Y}^Z_t P^Z_t - r_{t,t+1}\gamma_n\mathbb{E}_t\left(\frac{\widetilde{n}_{t+1}}{\widetilde{n}_t}-1\right)\left(\frac{\widetilde{n}_{t+1}}{\widetilde{n}^2_t}\right)\widetilde{Y}^Z_{t+1}P^Z_{t+1}\right\}
$$

Substituting (85) and (86) into (84) we obtain an expression for the real marginal cost in units of the wholesale domestic good:

$$
\begin{aligned}
mc^Z_t = \frac{1}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}\frac{\left(R^k_t\right)^{\alpha}}{z_t A^{1-\alpha}_t}\Bigg\{ & W_t + \gamma_n\left(\frac{\widetilde{n}_t}{\widetilde{n}_{t-1}}-1\right)\left(\frac{1}{\widetilde{n}_{t-1}}\right)\widetilde{Y}^Z_t P^Z_t \\
& - r_{t,t+1}\gamma_n\mathbb{E}_t\left(\frac{\widetilde{n}_{t+1}}{\widetilde{n}_t}-1\right)\left(\frac{\widetilde{n}_{t+1}}{\widetilde{n}^2_t}\right)\widetilde{Y}^Z_{t+1}P^Z_{t+1}\Bigg\}^{1-\alpha}
\end{aligned}
$$

In a second stage, the wholesale firm maximize its profits from the production of Y^Z_t, which is sold as X^Z_t at P^Z_t. The problem is:

$$
\max_{Y^Z_t}\left(P^Z_t - mc^Z_t\right)Y^Z_t
$$

The first-order condition implies that

$$P_t^Z = mc_t^Z.$$

A.4.7 Foreign composite goods

As in the case of home composite goods, a representative foreign composite goods firm demands foreign goods of all varieties $j \in [0,1]$ in amounts X_{jt}^F and combines them according to the technology

$$Y_t^F = \left[\int_0^1 \left(X_{jt}^F\right)^{\frac{\epsilon_F - 1}{\epsilon_F}} dj\right]^{\frac{\epsilon_F}{\epsilon_F - 1}} \tag{87}$$

with $\epsilon_F > 0$. Let P_{jt}^F denote the price of the foreign good of variety j. Analogously to the case of home composite goods, profit maximization yields the input demand functions

$$X_{jt}^F = \left(\frac{P_{jt}^F}{P_t^F}\right)^{-\epsilon_F} Y_t^F \tag{88}$$

for all j, and substituting (88) into (87) yields the price of foreign composite goods:

$$P_t^F = \left[\int_0^1 \left(P_{jt}^F\right)^{1-\epsilon_F} dj\right]^{\frac{1}{1-\epsilon_F}} \tag{89}$$

A.4.8 Foreign goods of variety j

Importing firms buy an amount M_t of a homogeneous foreign good at the price $P_t^{M\star}$ abroad and convert this good into varieties Y_{jt}^F that are sold domestically, and where total imports are $\int_0^1 Y_{jt}^F dj$. We assume that the import price level $P_t^{M\star}$ cointegrates with the foreign producer price level $P_t^\star$, i.e., $P_t^{M\star} = P_t^\star \xi_t^m$, where ξ_t^m is a stationary exogenous process. The firm producing variety j satisfies the demand given by (88) but it has monopoly power for its variety. As it takes one unit of the foreign good to produce one unit of variety j, nominal marginal costs in terms of composite goods prices are

$$P_t^F mc_{jt}^F = P_t^F mc_t^F = S_t P_t^{M\star} = S_t P_t^\star \xi_t^m \tag{90}$$

Given marginal costs, the firm producing variety j chooses its price P_{jt}^F to maximize profits. In setting prices, the firm faces a Calvo-type problem similar to domestic firms, whereby each period the firm can change its price optimally with probability $1-\theta_F$, and if it cannot optimally change its price, it indexes its previous price according to a weighted product of past and steady state inflation with weights $\kappa_F \in [0,1]$ and $1-\kappa_F$ respectively. A firm reoptimizing in period t will choose the price $\tilde{P}_{jt}^F$ that maximizes the current market value of the profits generated until it can reoptimize.[13] The solution to this problem is analogous to the case of domestic varieties, implying the first-order condition

$$F_t^{F_1} = F_t^{F_2} = F_t^F \tag{91}$$

where, defining $\tilde{p}_t^F = \tilde{P}_t^F / P_t^F$,

$$F_t^{F_1} = \frac{\epsilon_F - 1}{\epsilon_F}\left(\tilde{p}_t^F\right)^{1-\epsilon_F} Y_t^F + \theta_F E_t\left\{r_{t,t+1}\left(\frac{\tilde{p}_t^F \pi_{t+1}^{I,F}}{\tilde{p}_{t+1}^F}\right)^{1-\epsilon_F}\left(\pi_{t+1}^F\right)^{\epsilon_F} F_{t+1}^{F_1}\right\}$$

and

$$F_t^{F_2} = \left(\tilde{p}_t^F\right)^{-\epsilon_F} mc_t^F Y_t^F + \theta_F E_t\left\{r_{t,t+1}\left(\frac{\tilde{p}_t^F \pi_{t+1}^{I,F}}{\tilde{p}_{t+1}^F}\right)^{-\epsilon_F}\left(\pi_{t+1}^F\right)^{1+\epsilon_F} F_{t+1}^{F_2}\right\}$$

Using (89), we further have

$$1 = (1-\theta_F)\left(\tilde{p}_t^F\right)^{1-\epsilon_F} + \theta_F\left(\frac{\pi_t^{I,F}}{\pi_t^F}\right)^{1-\epsilon_F} \tag{92}$$

[13] As in the home varieties case, the following relation holds: $P_{jt+s}^F = \tilde{P}_{jt}^F \pi_{t+1}^{I,F} \dots \pi_{t+s}^{I,F}$, where $\pi_t^{I,F} = (\pi_{t-1}^F)^{\kappa_F}(\pi_t^T)^{1-\kappa_F}$, and, in turn, $\pi_t^F = P_t^F / P_{t-1}^F$.

A.4.9 Wages

Recall that demand for productive labor is satisfied by perfectly competitive packing firms that demand all varieties $i \in [0,1]$ of labor services in amounts $n_t(i)$ and combine them in order to produce composite labor services $\widetilde{n}_t$. The production function, variety i demand, and aggregate nominal wage are respectively given by:

$$\widetilde{n}_t = \left[\int_0^1 n_t(i)^{\frac{\epsilon_W - 1}{\epsilon_W}} di\right]^{\frac{\epsilon_W}{\epsilon_W - 1}}, \qquad \epsilon_W > 0. \tag{93}$$

$$n_t(i) = \left(\frac{W_t(i)}{W_t}\right)^{-\epsilon_W} \widetilde{n}_t \tag{94}$$

$$W_t = \left[\int_0^1 W_t(i)^{1-\epsilon_W} di\right]^{\frac{1}{1-\epsilon_W}}. \tag{95}$$

Regarding the supply of differentiated labor, as in Erceg et al. (2010), there is a continuum of monopolistically competitive unions indexed by $i \in [0,1]$, which act as wage setters for the differentiated labor services supplied by households. These unions allocate labor demand uniformly across patient and impatient households, so $n_t^P(i) = n_t^I(i)$ and $n_t^P(i) + n_t^I(i) = n_t(i)\ \forall i, t$, with $n_t^P(i) = \wp_U n_t^U(i) + (1-\wp_U) n_t^R(i)$, which also holds for the aggregate n_t^P, n_t^I and n_t.

The union supplying variety i satisfies the demand given by (94) but it has monopoly power for its variety. Wage setting is subject to a Calvo-type problem, whereby each period a union can set its nominal wage optimally with probability $1-\theta_W$. The wages of unions that cannot optimally adjust, are indexed to a weighted average of past and steady state productivity and inflation, with a gross growth rate of

$$\pi_t^{I,W} \equiv a_{t-1}^{\alpha_W} a^{1-\alpha_W} \pi_{t-1}^{\kappa_W} \pi^{1-\kappa_W}$$

Where $\Gamma_{t,s}^W = \Pi_{i=1}^s \pi_{t+i}^{I,W}$ is the growth of indexed wages s periods ahead of t. A union reoptimizing in period t chooses the wage $\widetilde{W}_t$ (equal for patient and impatient households) that maximizes the households' discounted lifetime utility. This union weights the benefits of wage income by considering the agents' marginal utility of consumption –which will usually differ between patient and impatient households– and weighs each household equally by considering a lagrangian multiplier of $\lambda_t^W = \left(\lambda_t^P + \lambda_t^I\right)/2$, with $\lambda_t^P = \wp_U \lambda_t^U + (1-\wp_U)\lambda_t^R$. We assume, for the sake of simplicity, that $\beta_W = (\beta_P + \beta_I)/2$ with $\beta_P = \wp_U \beta_U + (1-\wp_U)\beta_R$, and $\Theta_t = \left(\Theta_t^P + \Theta_t^I\right)/2$ with $\Theta_t^P = \wp_U \Theta_t^U + (1-\wp_U)\Theta_t^R$.

All things considered, taking the aggregate nominal wage as given, the union i's maximization problem can be expressed as

$$\begin{aligned}
&\max_{\widetilde{W}_t(i)} E_t \sum_{s=0}^{\infty} (\beta_U \theta_W)^s \varrho_{t+s} \left(\frac{\lambda_{t+s}^U A_{t+s}^{-\sigma}}{P_{t+s}} \widetilde{W}_t \Gamma_{t,s}^W n_{t+s}(i) - \Theta_{t+s}(A_{t+s})^{1-\sigma} \xi_{t+s}^n \frac{n_{t+s}(i)^{1+\varphi}}{1+\varphi}\right), \\
&\text{s.t.} \quad n_{t+s}(i) = \left(\frac{\widetilde{W}_t \Gamma_{t,s}^W}{W_{t+s}}\right)^{-\epsilon_W} \widetilde{n}_{t+s},
\end{aligned}$$

Which, after some derivation, results in the FOCs in a recursive formulation:

$$f_t^{W1} = \tilde{w}_t^{1-\epsilon_W} \left(\frac{\epsilon_W - 1}{\epsilon_W}\right) \widetilde{n}_t + \beta_U \theta_W \mathbb{E}_t \left\{ a_{t+1}^{-\sigma} \frac{\varrho_{t+1}}{\varrho_t} \frac{\lambda_{t+1}^U}{\lambda_t^U} \frac{\pi_{t+1}^W}{\pi_{t+1}} \left(\frac{\pi_{t+1}^{\widetilde{W}}}{\pi_{t+1}^{I,W}}\right)^{\epsilon_W - 1} f_{t+1}^{W1} \right\}$$

$$f_t^{W2} = \tilde{w}_t^{-\epsilon_W(1+\varphi)} mc_t^W \widetilde{n}_t + \beta_U \theta_W \mathbb{E}_t \left\{ a_{t+1}^{-\sigma} \frac{\varrho_{t+1}}{\varrho_t} \frac{\lambda_{t+1}^U}{\lambda_t^U} \frac{\pi_{t+1}^W}{\pi_{t+1}} \left(\frac{\pi_{t+1}^{\widetilde{W}}}{\pi_{t+1}^{I,W}}\right)^{\epsilon_W(1+\varphi)} f_{t+1}^{W2} \right\}$$

Where $f_t^{W1} = f_t^{W2} = f_t^W$ are the LHS and RHS of the FOC respectively, $mc_t^W = -{}^{(U_n/U_C)}/_{(W_t/A_t P_t)} = {}^{\xi_t^n (\widetilde{n}_t)^{\varphi}}/_{\lambda_t^U}\left(\frac{A_t P_t}{W_t}\right)\Theta_t$, is the gap with the efficient allocation when wages are flexible[14], $\pi_{t+1}^W = {}^{W_{t+1}}/_{W_t}$, $\pi_{t+1}^{\widetilde{W}} = {}^{\widetilde{W}_{t+1}}/_{\widetilde{W}_t}$ and $\tilde{w}_t = \tilde{W}_t / W_t$.

Further, let $\Psi^W(t)$ denote the set of labor markets in which wages are not reoptimized in period t. By (95), the aggregate wage index W_t evolves as follows:

$$\begin{aligned}
(W_t)^{1-\epsilon_W} = \int_0^1 W_t(i)^{1-\epsilon_W} di &= (1-\theta_W)\left(\widetilde{W}_t\right)^{1-\epsilon_W} + \int_{\Psi^W(t)} \left[W_{t-1}(i)\pi_t^{I,W}\right]^{1-\epsilon_W} di, \\
&= (1-\theta_W)\left(\widetilde{W}_t\right)^{1-\epsilon_W} + \theta_W \left[W_{t-1}\pi_t^{I,W}\right]^{1-\epsilon_W},
\end{aligned}$$

[14] U_n and U_C are the first derivatives of the utility function with respect to labor and consumption respectively.

or, dividing both sides by $(W_t)^{1-\epsilon_W}$:

$$1 = (1-\theta_W)\tilde{w}_t^{1-\epsilon_W} + \theta_W \left(\frac{\pi_t^{I,W}}{\pi_t^W}\right)^{1-\epsilon_W}.$$

The third equality above follows from the fact that the distribution of wages that are not reoptimized in period t corresponds to the distribution of effective wages in period $t-1$, though with total mass reduced to θ_W.

Finally, the clearing condition for the labor market is

$$n_t = \int_0^1 n_t(i)\,di = \widetilde{n}_t \int_0^1 \left(\frac{W_t(i)}{W_t}\right)^{-\epsilon_W} di = \widetilde{n}_t \Xi_t^W,$$

Where Ξ_t^W is a wage dispersion term that satisfies

$$\Xi_t^W = (1-\theta_W)\tilde{w}_t^{-\epsilon_W} + \theta_W \left(\frac{\pi_t^{I,W}}{\pi_t^W}\right)^{-\epsilon_W} \Xi_{t-1}^W.$$

A.4.10 Commodities

We assume the country receives an exogenous and stochastic endowment of commodities Y_t^{Co}. Moreover, these commodities are not consumed domestically but entirely exported. Therefore, the entire production is sold at a given international price $P_t^{Co\star}$, which is assumed to evolve exogenously. We further assume that the government receives a share $\chi \in [0,1]$ of this income and the remaining share goes to foreign agents.

A.5 Fiscal and monetary policy

The government consumes an exogenous stream of final goods G_t, pays through an insurance agency IA_t for deposits and bonds defaulted by banks, levies lump-sum taxes on patient households T_t^P, and issues one-period bonds BS_t^G and long-term bonds BL_t^G. Hence, the government satisfies the following period-by-period constraint:

$$T_t - BS_t^G - Q_t^{BL} BL_t^G + \chi S_t P_t^{Co\star} Y_t^{Co} = P_t G_t - R_{t-1} BS_{t-1}^G - R_t^{BL} Q_t^{BL} BL_{t-1}^G + IA_t \tag{96}$$

where

$$T_t = \alpha^T GDPN_t + \epsilon_t \left(BS_{SS}^G - BS_t^G + Q_{SS}^{BL} BL_{SS}^G - Q_t^{BL} BL_t^G\right) \tag{97}$$

and

$$IA_t = \gamma_D PD_t^D R_{t-1}^D D_{t-1}^F + \gamma_{BH} PD_t^H R_t^{BB} Q_t^{BB} BB_{t-1}^{Pr} \tag{98}$$

As in Chen et al. (2012), we assume that the government control the supply of long-term bonds according to a simple rule given by an exogenous AR(1) process on BL_t^G. In turn, monetary policy is carried out according to a Taylor-type rule of the form

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\alpha_R} \left[\left(\frac{(1-\alpha_E)\pi_t + \alpha_E \mathbb{E}_t\{\pi_{t+4}\}}{\pi_t^T}\right)^{\alpha_\pi} \left(\frac{GDP_t/GDP_{t-1}}{a}\right)^{\alpha_y}\right]^{1-\alpha_R} e_t^m \tag{99}$$

where $\alpha_R \in [0,1)$, $\alpha_\pi > 1$, $\alpha_y \geq 0$, $\alpha_E \in [0,1]$ and where π_t^T is an exogenous inflation target and e_t^m an i.i.d. shock that captures deviations from the rule.[15]

A.6 Rest of the world

Foreign agents demand home composite goods and buy the domestic commodity production. There are no transaction costs or other barriers to trade. The structure of the foreign economy is identical to the domestic economy, but the domestic economy is assumed to be small relative to the foreign economy. The latter implies that the foreign producer price level $P_t^\star$ is identical to the foreign consumption-based price index. Further, let $P_t^{H\star}$ denote the price of home composite goods expressed in foreign currency. Given full tradability and competitive export pricing, the law of one price holds separately for home composite goods and the commodity good, i.e. $P_t^H = S_t P_t^{H\star}$ and $P_t^{Co} = S_t P_t^{Co\star}$. That is, domestic and foreign prices of both goods are identical when expressed in the same currency. Due to local currency pricing, a weak form of the law of one price holds for foreign composite goods, i.e., $P_t^F mc_t^F = S_t P_t^\star \xi_t^m$ from (90). The real exchange rate rer_t therefore satisfies

$$rer_t = \frac{S_t P_t^\star}{P_t} = \frac{P_t^F}{P_t}\frac{mc_t^F}{\xi_t^m} \tag{100}$$

[15] We do not need a time-varying target, so we will set it to a constant.

We also have the following relation

$$\frac{rer_t}{rer_{t-1}} = \frac{\pi_t^s \pi_t^\star}{\pi_t} \tag{101}$$

where $\pi_t^s = S_t/S_{t-1}$. Foreign demand for the home composite good $X_t^{H\star}$ is given by

$$X_t^{H\star} = \left(\frac{P_t^H}{S_t P_t^\star}\right)^{-\eta^\star} Y_t^\star \tag{102}$$

with $\eta^\star > 0$ and where $Y_t^\star$ denotes foreign aggregate demand or GDP. Both $Y_t^\star$ and $\pi_t^\star$ evolve exogenously. The relevant foreign nominal interest rate is composed by an exogenous risk-free world interest rate R_t^W plus a country premium that decreases with the economy's net foreign asset position (expressed as a ratio of nominal GDP):

$$R_t^\star = R_t^W \exp\left\{-\frac{\phi^\star}{100}\left(\frac{S_t B_t^\star}{GDPN_t} - \bar{b}\right)\right\} \xi_t^R z_t^R \tag{103}$$

with $\phi^\star > 0$ and where ξ_t^R is an exogenous shock to the country premium.

A.6.1 Aggregation across patient households

Aggregate variables add up the per-capita amounts from unrestricted and restricted patient households, according to their respective mass $\wp_U$ and $1 - \wp_U$:

$$\begin{aligned}
C_t^P &= \wp_U C_t^U + (1-\wp_U) C_t^R \\
H_t^P &= \wp_U H_t^U + (1-\wp_U) H_t^R \\
n_t^P &= \wp_U n_t^U + (1-\wp_U) n_t^R \\
n_t^U &= n_t^R \\
D_t^{Tot} &= \wp_U D_t^U \\
B_t^{*,Tot} &= \wp_U B_t^{\star,U} \\
BS_t^{Pr} &= \wp_U BS_t^U \\
BL_t^{Pr} &= \wp_U BL_t^U + (1-\wp_U) BL_t^R \\
BB_t^{Pr} &= \wp_U BB_t^U
\end{aligned}$$

A.6.2 Goods market clearing

In the market for the final good, the clearing condition is

$$Y_t^C = C_t^P + C_t^I + I_t + I_t^H + G_t + \Upsilon_t/P_t \tag{104}$$

where Υ_t includes final goods used in default costs: the resources lost by households recovering deposits at failed banks, the resources lost by the banks to recover the proceeds from defaulted bank loans by the recovery of deposits by the deposit insurance agency and the cost of adjusting labor.

$$\Upsilon_t = \begin{array}{c}
\gamma_D PD_t^B R_{t-1}^D D_{t-1}^{Tot} + \gamma_D PD_t^B Q_t^{BB} R_t^{BB} BB_{t-1}^{Pr} + \mu_e G_e(\bar{\omega}_t^e) R_t^e Q_{t-1}^K K_{t-1} + \mu_I G_I(\bar{\omega}_t^I) R_t^H Q_{t-1}^H H_{t-1}^I \\
+\mu_H G_H(\bar{\omega}_t^H) \tilde{R}_t^H Q_{t-1}^L L_{t-1}^H + \mu_F G_F(\bar{\omega}_t^F) \tilde{R}_t^F L_{t-1}^F \\
+\frac{\gamma_n}{2}\left(\frac{\tilde{n}_t}{\tilde{n}_{t-1}} - 1\right)^2 Y_t^Z + Q_t^L (L_t^H - \kappa L_{t-1}^H)\left[\frac{\gamma_L}{2}\left(\frac{L_t^H - \kappa L_{t-1}^H}{L_{t-1}^H - \kappa L_{t-2}^H} - \bar{a}\right)^2\right]
\end{array}$$

In the market for the home and foreign composite goods we have, respectively,

$$Y_t^H = X_t^H + X_t^{H\star} \tag{105}$$

and

$$Y_t^F = X_t^F \tag{106}$$

while in the market for home and foreign varieties we have, respectively,

$$Y_{jt}^H = X_{jt}^H \tag{107}$$

and

$$Y_{jt}^F = X_{jt}^F \tag{108}$$

for all j.

In the market for the wholesale domestic good, we have

$$Y_t^Z = X_t^Z \tag{109}$$

Finally, in the market for housing, demand from both households must equal supply from housing producers:

$$H_t = H_t^P + H_t^I \tag{110}$$

A.6.3 Factor market clearing

In the market for labor, the clearing conditions are:

$$n_t^P + n_t^I = n_t = \widetilde{n}_t \Xi_t^W \tag{111}$$

$$n_t^P = n_t^I = \frac{n_t}{2} \tag{112}$$

Combining (86) and (85), the capital-labor ratio satisfies:

$$\frac{K_{t-1}}{\widetilde{n}_t} = \frac{\alpha}{(1-\alpha) R_t^k} \left\{ W_t + \gamma_n \left(\frac{\widetilde{n}_t}{\widetilde{n}_{t-1}} - 1 \right) \left(\frac{1}{\widetilde{n}_{t-1}} \right) Y_t^Z P_t^Z - r_{t,t+1} \gamma_n \mathbb{E}_t \left(\frac{\widetilde{n}_{t+1}}{\widetilde{n}_t} - 1 \right) \left(\frac{\widetilde{n}_{t+1}}{\widetilde{n}_t^2} \right) Y_{t+1}^Z P_{t+1}^Z \right\} \tag{113}$$

A.6.4 Deposits clearing

Bank F takes deposits, and its demand must equal the supply from unrestricted households:

$$D_t^F = D_t^{Tot} \tag{114}$$

A.6.5 Domestic bonds clearing

The aggregate net holding of participating agents in bond markets are in zero net supply:

$$BL_t^{Pr} + BL_t^{CB} + BL_t^G = 0 \tag{115}$$

$$BS_t^{Pr} + BS_t^G = 0 \tag{116}$$

Where BL_t^{CB} is an exogenous process that represents the long-term government bond purchases done by the Central Bank.

A.6.6 No-arbitrage condition in bond markets

The no-arbitrage condition implies the following relation between short and long-tem interest rates:

$$R_t \left(\frac{1+\zeta_t^L}{R_t^{BL} - \kappa_B} \right) = \mathbb{E}_t \left\{ \frac{\varrho_{t+1} \lambda_{t+1}^{UP}}{\pi_{t+1}} \left(\frac{R_{t+1}^{BL}}{R_{t+1}^{BL} - \kappa_B} \right) A_{t+1}^{-\sigma} \right\} \left(\mathbb{E}_t \left\{ \frac{\varrho_{t+1} \lambda_{t+1}^{UP}}{\pi_{t+1}} A_{t+1}^{-\sigma} \right\} \right)^{-1}$$

which can be further rearranged (up to a first order) by using the definition of R_t^{BL}

$$R_t \left(1 + \zeta_t^L \right) \quad \approx \quad \mathbb{E}_t \left\{ \left(\frac{Q_{t+1}^{BL}}{Q_t^{BL}} R_{t+1}^{BL} \right) \right\} \tag{117}$$

A.6.7 Inflation and relative prices

The following holds for $j = H, F$:

$$p_t^j = \frac{P_t^j}{P_t}$$

and, also,

$$\frac{p_t^j}{p_{t-1}^j} = \frac{\pi_t^j}{\pi_t}$$

A.6.8 Aggregate supply

Using the productions of different varieties of home goods (78)

$$\int_0^1 Y_{jt}^H dj = X_t^Z$$

Integrating (107) over j and using (76) then yields aggregate output of home goods as

$$\int_0^1 Y_{jt}^H dj = \int_0^1 X_{jt}^H dj = Y_t^H \int_0^1 \left(p_{jt}^H\right)^{-\epsilon_H} dj$$

or, combining the previous two equations,

$$Y_t^H \Xi_t^H = X_t^Z$$

where Ξ_t^H is a price dispersion term satisfying

$$\begin{aligned} \Xi_t^H &= \int_0^1 \left(\frac{P_{jt}^H}{P_t^H}\right)^{-\epsilon_H} dj \\ &= (1-\theta_H)\left(\tilde{p}_t^H\right)^{-\epsilon_H} + \theta_H \left(\frac{\pi_t^{I,H}}{\pi_t^H}\right)^{-\epsilon_H} \Xi_{t-1}^H \end{aligned}$$

A.6.9 Aggregate demand

Aggregate demand or GDP is defined as the sum of domestic absorption and the trade balance. Domestic absorption is equal to $Y_t^C = C_t^P + C_t^I + I_t + I_t^H + G_t + \Upsilon_t$. The nominal trade balance is defined as

$$TB_t = P_t^H X_t^{H\star} + S_t P_t^{Co\star} Y_t^{Co} - S_t P_t^{M\star} M_t \tag{118}$$

Integrating (108) over j and using (88) allows us to write imports as

$$M_t = \int_0^1 Y_{jt}^F dj = \int_0^1 X_{jt}^F dj = Y_t^F \int_0^1 \left(\frac{P_{jt}^F}{P_t^F}\right)^{-\epsilon_F} dj = Y_t^F \Xi_t^F$$

where Ξ_t^F is a price dispersion term satisfying

$$\Xi_t^F = (1-\theta_F)\left(\tilde{p}_t^F\right)^{-\epsilon_F} + \theta_F \left(\frac{\pi_t^{I,F}}{\pi_t^F}\right)^{-\epsilon_F} \Xi_{t-1}^F$$

We then define real GDP as

$$GDP_t = Y_t^{NoCo} + Y_t^{Co}$$

where non-mining GDP, Y_t^{NoCo}, is given by

$$Y_t^{NoCo} = C_t^P + C_t^I + I_t + I_t^H + G_t + X_t^{H\star} - M_t$$

and nominal GDP is defined as

$$GDPN_t = P_t \left(C_t^P + C_t^I + I_t + I_t^H + G_t\right) + TB_t \tag{119}$$

Note that by combining (119) with the zero profit condition in the final goods sector, i.e., $P_t Y_t^C = P_t^H X_t^H + P_t^F X_t^F$, and using the market clearing conditions for final and composite goods, (104)-(105), GDP is seen to be equal to total value added (useful for the steady state):

$$\begin{aligned} GDPN_t &= P_t Y_t^C - \Upsilon_t + P_t^H X_t^{H\star} + S_t P_t^{Co\star} Y_t^{Co} - S_t P_t^{M\star} M_t \\ &= P_t^H X_t^H + P_t^F X_t^F - \Upsilon_t + P_t^H X_t^{H\star} + S_t P_t^{Co\star} Y_t^{Co} - S_t P_t^{M\star} M_t \\ &= P_t^H Y_t^H + S_t P_t^{Co\star} Y_t^{Co} + P_t^F X_t^F - S_t P_t^{M\star} M_t - \Upsilon_t \end{aligned}$$

A.6.10 Balance of payments

Aggregate nominal profits, dividends, rents and taxes are given by

$$
\begin{aligned}
\Psi_t \;=\;& \underbrace{P_t Y_t^C - P_t^H X_t^H - P_t^F X_t^F}_{\Pi_t^C} + \underbrace{P_t^H Y_t^H - \int_0^1 P_{jt}^H X_{jt}^H dj}_{\Pi_t^H} + \underbrace{P_t^F Y_t^F - \int_0^1 P_{jt}^F X_{jt}^F dj}_{\Pi_t^F} \\
& + \underbrace{\int_0^1 Y_{jt}^H \left(P_{jt}^H - P_t^Z\right) dj}_{\int_0^1 \Pi_{jt}^H dj} + \underbrace{\int_0^1 \left(P_{jt}^F Y_{jt}^F - S_t P_t^{M\star} Y_{jt}^F\right) dj}_{\int_0^1 \Pi_{jt}^F dj} \\
& + \underbrace{Q_t^K \left(K_t - \left(1-\delta_K\right) K_{t-1}\right) - P_t I_t}_{\Pi_t^I} + \underbrace{Q_t^H \left(H_t - \left(1-\delta_H\right) H_{t-1}\right) - P_t I_t^H}_{\Pi_t^{I^H}} + \underbrace{\left(P_t^Z - mc_t^Z\right) Y_t^Z}_{\Pi_t^Z} \\
& + \underbrace{\zeta_t^L \left(\frac{1}{R_t^{BL} - \kappa_B}\right) BL_t^U}_{\Pi_t^F} + C_t^e + C_t^b + S_t REN_t^* - T_t \\
=\;& P_t \left(C_t + G_t\right) + \Upsilon_t + P_t^H X_t^{H\star} - S_t P_t^{M\star} M_t - W_t n_t - R_t^k K_{t-1} + Q_t^K \left(K_t - \left(1-\delta_K\right) K_{t-1}\right) \\
& + Q_t^H \left(H_t - \left(1-\delta_H\right) H_{t-1}\right) + C_t^e + C_t^b + S_t REN_t^* - T_t + \zeta_t^L \left(\frac{1}{R_t^{BL} - \kappa_B}\right) BL_t^U \\
=\;& P_t \left(C_t + G_t\right) + \Upsilon_t + TB_t - S_t P_t^{Co\star} Y_t^{Co} - W_t n_t - R_t^k K_{t-1} + Q_t^K \left(K_t - \left(1-\delta_K\right) K_{t-1}\right) \\
& + Q_t^H \left(H_t - \left(1-\delta_H\right) H_{t-1}\right) + C_t^e + C_t^b + S_t REN_t^* - T_t + \zeta_t^L \left(\frac{1}{R_t^{BL} - \kappa_B}\right) BL_t^U
\end{aligned}
$$

Where the second equality uses the market clearing conditions (104)-(116), and the third equality uses the definition of the trade balance, (118). Substituting out Ψ_t in the households' budget constraint (40) and using the government's budget constraint (96) to substitute out taxes T_t shows that the net foreign asset position evolves according to

$$
S_t B_t^\star = S_t B_{t-1}^\star R_{t-1}^\star + TB_t + S_t REN_t^* - \left(1-\chi\right) S_t P_t^{Co\star} Y_t^{Co}
$$

B Stationary Equilibrium Conditions

In the model described in the previous sections, real variables in uppercase contain a unit root in equilibrium due to the presence of the non-stationary productivity vector A_t. Uppercase nominal variables contain an additional unit root given by the non-stationarity of the price level. In this section we show the stationary version of the model, where we define $a_t = A_t/A_{t-1}$, and all lowercase variables denote the stationary counterpart of the original variables, obtained by dividing them by its co-integration vector(A_t or P_t).

The rational expectations equilibrium of the stationary version of the model is then the set of sequences for the endogenous variables such that for a given set of initial values and exogenous processes the following conditions are satisfied:

B.1 Patient Households

B.1.1 Unrestricted (U)

$$\hat{c}_t^U = \left[(1 - o_{\hat{C}})^{\frac{1}{\eta_{\hat{C}}}} \left(c_t^U - \phi_c \frac{c_{t-1}^U}{a_t} \right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}} + (o_{\hat{C}})^{\frac{1}{\eta_{\hat{C}}}} \left(\xi_t^h \left(\frac{h_{t-1}^U}{a_t} - \phi_{hh} \frac{h_{t-2}^U}{a_t a_{t-1}} \right) \right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}} \right]^{\frac{\eta_{\hat{C}}}{\eta_{\hat{C}}-1}} \tag{1}$$

$$\lambda_t^U = \left(\hat{c}_t^U \right)^{-\sigma} \left(\frac{(1 - o_{\hat{C}})\, \hat{c}_t^U}{\left(c_t^U - \phi_c \frac{c_{t-1}^U}{a_t} \right)} \right)^{\frac{1}{\eta_{\hat{C}}}} \tag{2}$$

$$\varrho_t \lambda_t^U q_t^H = \beta_U \mathbb{E}_t \varrho_{t+1} \left\{ \left(\hat{c}_{t+1}^U a_{t+1} \right)^{-\sigma} \xi_{t+1}^h \left(\frac{o_{\hat{C}} \hat{c}_{t+1}^U a_{t+1}}{\xi_{t+1}^h \left(h_t^U - \phi_{hh} \frac{h_{t-1}^U}{a_t} \right)} \right)^{\frac{1}{\eta_{\hat{C}}}} + (1 - \delta_H)\, \lambda_{t+1}^U a_{t+1}^{-\sigma} q_{t+1}^H \right\} \tag{3}$$

$$\varrho_t \lambda_t^U = \beta_U R_t \mathbb{E}_t \left\{ \frac{\varrho_{t+1} \lambda_{t+1}^U}{\pi_{t+1}} a_{t+1}^{-\sigma} \right\} \tag{4}$$

$$\varrho_t \lambda_t^U = \beta_U \mathbb{E}_t \left\{ \frac{\tilde{R}_{t+1}^D}{\pi_{t+1}} \varrho_{t+1} \lambda_{t+1}^U a_{t+1}^{-\sigma} \right\} \tag{5}$$

$$\varrho_t \lambda_t^U = \beta_U R_t^\star \mathbb{E}_t \left\{ \frac{\varrho_{t+1} \lambda_{t+1}^U \pi_{t+1}^s}{\pi_{t+1}} a_{t+1}^{-\sigma} \right\} \tag{6}$$

$$\varrho_t \lambda_t^U \left(1 + \zeta_t^L \right) q_t^{BL} = \beta_U \mathbb{E}_t \left\{ \varrho_{t+1} \lambda_{t+1}^U a_{t+1}^{-\sigma} R_{t+1}^{BL} q_{t+1}^{BL} \right\} \tag{7}$$

$$\varrho_t \lambda_t^U \left(1 + \zeta_t^L \right) q_t^{BB} = \beta_U \mathbb{E}_t \left\{ \varrho_{t+1} \lambda_{t+1}^U a_{t+1}^{-\sigma} \tilde{R}_{t+1}^{BB} q_{t+1}^{BB} \right\} \tag{8}$$

B.1.2 Restricted (R)

$$\hat{c}_t^R = \left[(1 - o_{\hat{C}})^{\frac{1}{\eta_{\hat{C}}}} \left(c_t^R - \phi_c \frac{c_{t-1}^R}{a_t} \right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}} + (o_{\hat{C}})^{\frac{1}{\eta_{\hat{C}}}} \left(\xi_t^h \left(\frac{h_{t-1}^R}{a_t} - \phi_{hh} \frac{h_{t-2}^R}{a_t a_{t-1}} \right) \right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}} \right]^{\frac{\eta_{\hat{C}}}{\eta_{\hat{C}}-1}} \tag{9}$$

$$\lambda_t^R = \left(\hat{c}_t^R \right)^{-\sigma} \left(\frac{(1 - o_{\hat{C}})\, \hat{c}_t^R}{\left(c_t^R - \phi_c \frac{c_{t-1}^R}{a_t} \right)} \right)^{\frac{1}{\eta_{\hat{C}}}} \tag{10}$$

$$\varrho_t \lambda_t^R q_t^H = \beta_R \mathbb{E}_t \varrho_{t+1} \left\{ \left(\hat{c}_{t+1}^R a_{t+1} \right)^{-\sigma} \left(\frac{o_{\hat{C}} \hat{c}_{t+1}^R a_{t+1}}{\xi_{t+1}^h \left(h_t^R - \phi_{hh} \frac{h_{t-1}^R}{a_t} \right)} \right)^{\frac{1}{\eta_{\hat{C}}}} \xi_{t+1}^h + (1 - \delta_H)\, \lambda_{t+1}^R a_{t+1}^{-\sigma} q_{t+1}^H \right\} \tag{11}$$

$$\varrho_t \lambda_t^R q_t^{BL} = \beta_R \mathbb{E}_t \left\{ \varrho_{t+1} \lambda_{t+1}^R q_{t+1}^{BL} R_{t+1}^{BL} a_{t+1}^{-\sigma} \right\} \tag{12}$$

$$q_t^{BL} bl_t^R + c_t^R + q_t^H h_t^R = q_t^{BL} R_t^{BL} \frac{bl_{t-1}^R}{a_t} + w_t n_t^R + q_t^H (1 - \delta_H) \frac{h_{t-1}^R}{a_t} \tag{13}$$

B.2 Impatient Households

$$\frac{R_t^H}{\pi_t} = \frac{q_t^H (1 - \delta_H)}{q_{t-1}^H} \tag{14}$$

$$\hat{c}_t^I = \left[\left(1 - o_{\hat{C}}\right)^{\frac{1}{\eta_{\hat{C}}}} \left(c_t^I - \phi_c \frac{c_{t-1}^I}{a_t} \right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}} + \left(o_{\hat{C}}\right)^{\frac{1}{\eta_{\hat{C}}}} \left(\xi_t^h \left(\frac{h_{t-1}^I}{a_t} - \phi_{hh} \frac{h_{t-2}^I}{a_t a_{t-1}} \right) \right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}} \right]^{\frac{\eta_{\hat{C}}}{\eta_{\hat{C}}-1}} \tag{15}$$

$$\lambda_t^I = \left(\hat{c}_t^I\right)^{-\sigma} \left(\frac{\left(1 - o_{\hat{C}}\right) \hat{c}_t^I}{\left(c_t^I - \phi_c \frac{c_{t-1}^I}{a_t} \right)} \right)^{\frac{1}{\eta_{\hat{C}}}} \tag{16}$$

$$\hat{q}_t^L = \left(1 - \frac{\kappa l_{t-1}^H}{l_t^H a_t} \right) q_t^L + \left(\frac{\kappa l_{t-1}^H}{l_t^H a_t} \right) \hat{q}_{t-1}^L \tag{17}$$

$$\bar{\omega}_t^I = \frac{\hat{R}_t^I \hat{q}_t^L l_{t-1}^H}{R_t^H q_{t-1}^H h_{t-1}^I} \pi_t \tag{18}$$

$$R_t^I = \frac{1}{q_t^L} + \kappa \tag{19}$$

$$\hat{R}_t^I = \frac{1 + \kappa \hat{q}_{t-1}^L}{\hat{q}_t^L} \tag{20}$$

$$\varrho_t \lambda_t^I q_t^H = \mathbb{E}_t \left\{ \begin{array}{c} \beta_I \varrho_{t+1} \left(\left(\hat{c}_{t+1}^I a_{t+1}\right)^{-\sigma} \left(\frac{o_{\hat{C}} \hat{c}_{t+1}^I a_{t+1}}{\xi_{t+1}^h \left(h_t^I - \phi_{hh} \frac{h_{t-1}^I}{a_t} \right)} \right)^{\frac{1}{\eta_{\hat{C}}}} \xi_{t+1}^h + \lambda_{t+1}^I a_{t+1}^{-\sigma} \left[1 - \Gamma_I \left(\bar{\omega}_{t+1}^I \right) \right] \frac{R_{t+1}^H}{\pi_{t+1}} q_t^H \right) \\ + \varrho_t \lambda_t^H \left[1 - \Gamma_H \left(\bar{\omega}_{t+1}^H \right) \right] \left[\Gamma_I \left(\bar{\omega}_{t+1}^I \right) - \mu_I G_I \left(\bar{\omega}_{t+1}^I \right) \right] R_{t+1}^H q_t^H \end{array} \right\} \tag{21}$$

$$\beta_I = \mathbb{E}_t \left\{ \frac{\varrho_t \lambda_t^H \pi_{t+1}}{\varrho_{t+1} \lambda_{t+1}^I a_{t+1}^{-\sigma}} \left[1 - \Gamma_H \left(\bar{\omega}_{t+1}^H \right) \right] \frac{\left[\Gamma_I' \left(\bar{\omega}_{t+1}^I \right) - \mu_I G_I' \left(\bar{\omega}_{t+1}^I \right) \right]}{\Gamma_I' \left(\bar{\omega}_{t+1}^I \right)} \right\} \tag{22}$$

$$c_t^I + q_t^H h_t^I - q_t^L (l_t^H - \frac{\kappa l_{t-1}^H}{a_t}) \left[1 - \frac{\gamma_L}{2} (\nabla \tilde{l}_t - \bar{a})^2 \right] - \frac{\kappa l_{t-1}^H \hat{q}_{t-1}^L}{a_t} = \frac{w_t n_t}{2} + \left[1 - \Gamma_I \left(\bar{\omega}_t^I \right) \right] \frac{R_t^H q_{t-1}^H h_{t-1}^I}{a_t \pi_t} \tag{23}$$

$$\begin{gathered} \varrho_t q_t^L \left\{ \lambda_t^I \left[1 - \frac{\gamma_L}{2} (\nabla \tilde{l}_t - \bar{a})^2 \right] - \lambda_t^I \nabla \tilde{l}_t \gamma_L (\nabla \tilde{l}_t - \bar{a}) - \lambda_t^H \rho_{t+1}^H \phi_H \right\} = ... \\ ... \beta_I \mathbb{E}_t \left\{ \varrho_{t+1} \lambda_{t+1}^I a_{t+1}^{-\sigma} \left[\kappa q_{t+1}^L \left[1 - \frac{\gamma_L}{2} (\nabla \tilde{l}_{t+1} - \bar{a})^2 \right] - q_{t+1}^L \nabla \tilde{l}_{t+1} \gamma_L (\nabla \tilde{l}_{t+1} - \bar{a}) (\nabla \tilde{l}_{t+1} + \kappa) - \kappa \hat{q}_t^L \right] \right\} \end{gathered} \tag{24}$$

$$PD_t^I = F_I \left(\bar{\omega}_t^I \right) \tag{25}$$

B.3 Entrepreneurs

$$q_t^K k_t = n_t^e + l_t^F \tag{26}$$

$$\frac{R_t^e}{\pi_t} = \frac{r_t^k + (1 - \delta_K) q_t^K}{q_{t-1}^K} \tag{27}$$

$$\bar{\omega}_t^e = \frac{R_{t-1}^L l_{t-1}^F}{R_t^e q_{t-1}^K k_{t-1}} \tag{28}$$

$$c_t^e = \chi_e \xi_t^{\chi_e} \psi_t^e \tag{29}$$

$$n_t^e = \left(1 - \chi_e \xi_t^{\chi_e}\right) \psi_t^e \tag{30}$$

$$\psi_t^e a_t \pi_t = \left[1 - \Gamma_e \left(\bar{\omega}_t^e\right)\right] R_t^e q_{t-1}^K k_{t-1} \tag{31}$$

$$\left(1 - \Gamma_{t+1}^e\right) = \lambda_t^e \left(\frac{\rho_{t+1}^F \phi_t^F}{R_{t+1}^e} - \left(1 - \Gamma_{t+1}^F\right) \left[\Gamma_{t+1}^e - \mu^e G_{t+1}^e\right] \right) \tag{32}$$

$$\Gamma_{t+1}^{e'} = \lambda_t^e \left(1 - \Gamma_{t+1}^F\right) \left[\Gamma_{t+1}^{e'} - \mu^e G_{t+1}^{e'}\right] \tag{33}$$

$$PD_t^e = F_e \left(\bar{\omega}_t^e\right) \tag{34}$$

B.4 Bankers and Banking System

$$\mathbb{E} \left[\rho_{t+1}^F \right] = \xi_t^{b,roe} \mathbb{E} \left[\tilde{\rho}_{t+1}^H \right] \tag{35}$$

$$c_t^b = \xi_t^{\chi_b} \chi_b \psi_t^b \tag{36}$$

$$n_t^b = \left(1 - \xi_t^{\chi_b}\chi_b\right)\psi_t^b \tag{37}$$

$$\psi_t^b a_t \pi_t = \rho_t^F e_{t-1}^F + \tilde{\rho}_t^H e_{t-1}^H \tag{38}$$

$$n_t^b = e_t^F + e_t^H \tag{39}$$

$$PD_t^D = \frac{Q_{t-1}^{BB} BB_{t-1} PD_t^H + d_{t-1}^{Tot} PD_t^F}{Q_{t-1}^{BB} BB_{t-1} + d_{t-1}^{Tot}} \tag{40}$$

B.5 F Banks

$$d_t^F + e_t^F = l_t^F \tag{41}$$

$$\bar{\omega}_t^F = \left(1 - \phi_{F,t-1}\right)\frac{R_{t-1}^D}{\tilde{R}_t^F} \tag{42}$$

$$e_t^F = \phi_{F,t} l_t^F \tag{43}$$

$$\rho_t^F = \left[1 - \Gamma_F\left(\bar{\omega}_t^F\right)\right]\frac{\tilde{R}_t^F}{\phi_{F,t-1}} \tag{44}$$

$$\tilde{R}_t^F = \left[\Gamma_e\left(\bar{\omega}_t^e\right) - \mu_e G_e\left(\bar{\omega}_t^e\right)\right]\frac{R_t^e q_{t-1}^K k_{t-1}}{l_{t-1}^F} \tag{45}$$

$$PD_t^F = F_F\left(\bar{\omega}_t^F\right) \tag{46}$$

B.6 H Banks

$$q_t^{BB} bb_t^{Pr} + e_t^H = \hat{q}_t^L l_t^H \tag{47}$$

$$\bar{\omega}_t^H = \left(1 - \phi_{H,t-1}\right)\frac{R_t^{BB} q_t^{BB}}{\tilde{R}_t^H q_{t-1}^{BB}}\pi_t \tag{48}$$

$$e_t^H = \phi_H \hat{q}_t^L l_t^H \tag{49}$$

$$\rho_t^H = \left[1 - \Gamma_H\left(\bar{\omega}_t^H\right)\right]\frac{\tilde{R}_t^H}{\phi_{H,t-1}} \tag{50}$$

$$\tilde{\rho}_t^H = (1-\kappa)\,\rho_t^H + \kappa\mathbb{E}\left[\tilde{\rho}_{t+1}^H\right] \tag{51}$$

$$\tilde{R}_t^H = \left[\Gamma_I\left(\bar{\omega}_t^I\right) - \mu_I G_I\left(\bar{\omega}_t^I\right)\right]\frac{R_t^H q_{t-1}^H h_{t-1}^I}{\hat{q}_{t-1}^L l_{t-1}^H} \tag{52}$$

$$PD_t^H = F_H\left(\bar{\omega}_t^H\right) \tag{53}$$

B.7 Capital and Housing Goods

$$k_t = (1-\delta_K)\frac{k_{t-1}}{a_t} + \left[1 - \frac{\gamma_K}{2}\left(\frac{i_t}{i_{t-1}}a_t - a\right)^2\right]\xi_t^i i_t \tag{54}$$

$$\begin{aligned} 1 \quad &= \quad q_t^K\left[1 - \frac{\gamma_K}{2}\left(\frac{i_t}{i_{t-1}}a_t - a\right)^2 - \gamma_K\left(\frac{i_t}{i_{t-1}}a_t - a\right)\frac{i_t}{i_{t-1}}a_t\right]\xi_t^i \\ &+ \quad \beta_P\mathbb{E}_t\left\{\frac{\varrho_{t+1}\lambda_{t+1}^P}{\varrho_t\lambda_t^P}a_{t+1}^{-\sigma}q_{t+1}^K\gamma_K\left(\frac{i_{t+1}}{i_t}a_{t+1} - a\right)\left(\frac{i_{t+1}}{i_t}a_{t+1}\right)^2\xi_{t+1}^i\right\} \end{aligned} \tag{55}$$

$$h_t = (1-\delta_H)\frac{h_{t-1}}{a_t} + \left[1 - \frac{\gamma_H}{2}\left(\frac{i_{t-N_H}^{AH}}{i_{t-N_H-1}^{AH}}a_t - a\right)^2\right]\xi_{t-N_H}^{ih}\frac{i_{t-N_H}^{AH}}{\prod_{i=0}^{N_H-1}a_{t-j}} \tag{56}$$

$$
\begin{aligned}
0 \;=\;& E_t \sum_{j=0}^{N_H} \beta_P^j \varrho_{t+j} \lambda_{t+j}^P \varphi_j^H \prod_{i=j+1}^{N_H} \left(a_{t+i}^{\sigma}\right) \\
-\;& E_t \beta_P^{N_H} \varrho_{t+N_H} \lambda_{t+N_H}^P q_{t+N_H}^H \left\{ \left[1 - \frac{\gamma_H}{2} \left(\frac{i_t^{AH}}{i_{t-1}^{AH}} a_t - a \right)^2 \right] - \gamma_H \left(\frac{i_t^{AH}}{i_{t-1}^{AH}} a_t - a \right) \frac{i_t^{AH}}{i_{t-1}^{AH}} a_t \right\} \xi_t^{ih} \\
-\;& E_t \beta_P^{N_H+1} \varrho_{t+N_H+1} \lambda_{t+N_H+1}^P q_{t+N_H+1}^H a_{t+N_H+1}^{-\sigma} \left\{ \gamma_H \left(\frac{i_{t+1}^{AH}}{i_t^{AH}} a_{t+1} - a \right) \left(\frac{i_{t+1}^{AH}}{i_t^{AH}} a_{t+1} \right)^2 \xi_{t+1}^{ih} \right\}
\end{aligned} \tag{57}
$$

$$
i_t^H = \sum_{j=0}^{N_H} \varphi_j^H \frac{i_{t-j}^{AH}}{\prod_{i=0}^{j-1} a_{t-j}} \tag{58}
$$

B.8 Final Goods

$$
y_t^C = \left[\omega^{1/\eta} \left(x_t^H \right)^{1-1/\eta} + (1-\omega)^{1/\eta} \left(x_t^F \right)^{1-1/\eta} \right]^{\frac{\eta}{\eta-1}} \tag{59}
$$

$$
x_t^F = (1-\omega) \left(p_t^F \right)^{-\eta} y_t^C \tag{60}
$$

$$
x_t^H = \omega \left(p_t^H \right)^{-\eta} y_t^C \tag{61}
$$

B.9 Home Goods

$$
f_t^H = \frac{\epsilon_H - 1}{\epsilon_H} \left(\tilde{p}_t^H \right)^{1-\epsilon_H} y_t^H + \beta_U \theta_H \mathbb{E}_t \left\{ \frac{\varrho_{t+1} \lambda_{t+1}^P a_{t+1}^{1-\sigma}}{\varrho_t \lambda_t^P \pi_{t+1}} \left(\frac{\tilde{p}_t^H \pi_{t+1}^{I,H}}{\tilde{p}_{t+1}^H} \right)^{1-\epsilon_H} \left(\pi_{t+1}^H \right)^{\epsilon_H} f_{t+1}^H \right\} \tag{62}
$$

$$
f_t^H = \left(\tilde{p}_t^H \right)^{-\epsilon_H} mc_t^H y_t^H + \beta_U \theta_H \mathbb{E}_t \left\{ \frac{\varrho_{t+1} \lambda_{t+1}^P a_{t+1}^{1-\sigma}}{\varrho_t \lambda_t^P \pi_{t+1}} \left(\frac{\tilde{p}_t^H \pi_{t+1}^{I,H}}{\tilde{p}_{t+1}^H} \right)^{-\epsilon_H} \left(\pi_{t+1}^H \right)^{1+\epsilon_H} f_{t+1}^H \right\} \tag{63}
$$

$$
1 = (1-\theta_H) \left(\tilde{p}_t^H \right)^{1-\epsilon_H} + \theta_H \left(\frac{\pi_t^{I,H}}{\pi_t^H} \right)^{1-\epsilon_H} \tag{64}
$$

$$
\pi_t^{I,H} = \left(\pi_{t-1}^H \right)^{\kappa_H} \left(\pi^T \right)^{1-\kappa_H} \tag{65}
$$

$$
mc_t^H = \frac{p_t^Z}{p_t^H} \tag{66}
$$

B.10 Wholesale Domestic Goods

$$
\begin{aligned}
mc_t^Z = \frac{1}{\alpha^\alpha (1-\alpha)^{1-\alpha}} \frac{(r_t^k)^\alpha}{z_t} \Bigg\{ & w_t + \gamma_n \left(\frac{\widetilde{n}_t}{\widetilde{n}_{t-1}} - 1 \right) \left(\frac{1}{\widetilde{n}_{t-1}} \right) y_t^Z p_t^Z \\
& - \beta_U \frac{\varrho_{t+1} \lambda_{t+1}^P a_{t+1}^{1-\sigma}}{\varrho_t \lambda_t^P} \gamma_n \mathbb{E}_t \left(\frac{\widetilde{n}_{t+1}}{\widetilde{n}_t} - 1 \right) \left(\frac{\widetilde{n}_{t+1}}{\widetilde{n}_t^2} \right) y_{t+1}^Z p_{t+1}^Z \Bigg\}^{1-\alpha}
\end{aligned} \tag{67}
$$

$$
\begin{aligned}
\frac{k_{t-1}}{\widetilde{n}_t} = \frac{\alpha}{(1-\alpha) r_t^k} \Bigg\{ & w_t + \gamma_n \left(\frac{\widetilde{n}_t}{\widetilde{n}_{t-1}} - 1 \right) \left(\frac{1}{\widetilde{n}_{t-1}} \right) y_t^Z p_t^Z \\
& - \beta_U \frac{\varrho_{t+1} \lambda_{t+1}^P a_{t+1}^{1-\sigma}}{\varrho_t \lambda_t^P} \gamma_n \mathbb{E}_t \left(\frac{\widetilde{n}_{t+1}}{\widetilde{n}_t} - 1 \right) \left(\frac{\widetilde{n}_{t+1}}{\widetilde{n}_t^2} \right) y_{t+1}^Z p_{t+1}^Z \Bigg\} a_t
\end{aligned} \tag{68}
$$

$$
p_t^Z = mc_t^Z \tag{69}
$$

B.11 Foreign Goods

$$
p_t^F mc_t^F = rer_t \xi_t^m \tag{70}
$$

$$
f_t^F = \frac{\epsilon_F - 1}{\epsilon_F} \left(\tilde{p}_t^F \right)^{1-\epsilon_F} y_t^F + \beta_U \theta_F \mathbb{E}_t \left\{ \frac{\varrho_{t+1} \lambda_{t+1}^P a_{t+1}^{1-\sigma}}{\varrho_t \lambda_t^P \pi_{t+1}} \left(\frac{\tilde{p}_t^F \pi_{t+1}^{I,F}}{\tilde{p}_{t+1}^F} \right)^{1-\epsilon_F} \left(\pi_{t+1}^F \right)^{\epsilon_F} f_{t+1}^F \right\} \tag{71}
$$

$$f_t^F = \left(\tilde{p}_t^F\right)^{-\epsilon_F} mc_t^F y_t^F + \beta_U \theta_F \mathbb{E}_t \left\{ \frac{\varrho_{t+1} \lambda_{t+1}^P a_{t+1}^{1-\sigma}}{\varrho_t \lambda_t^P \pi_{t+1}} \left(\frac{\tilde{p}_t^F \pi_{t+1}^{I,F}}{\tilde{p}_{t+1}^F} \right)^{-\epsilon_F} \left(\pi_{t+1}^F\right)^{1+\epsilon_F} f_{t+1}^F \right\} \tag{72}$$

$$1 = (1-\theta_F)\left(\tilde{p}_t^F\right)^{1-\epsilon_F} + \theta_F \left(\frac{\pi_t^{I,F}}{\pi_t^F} \right)^{1-\epsilon_F} \tag{73}$$

$$\pi_t^{I,F} = \left(\pi_{t-1}^F\right)^{\kappa_F} \left(\pi^T\right)^{1-\kappa_F} \tag{74}$$

B.12 Wages

$$\lambda_t^W = \frac{\lambda_t^P + \lambda_t^I}{2} \tag{75}$$

$$\lambda_t^P = \wp_U \lambda_t^U + (1-\wp_U)\lambda_t^R \tag{76}$$

$$\Theta_t = \frac{\left(\wp_U \Theta_t^U + (1-\wp_U)\Theta_t^R\right) + \Theta_t^I}{2} \tag{77}$$

$$mc_t^W = \Theta_t \frac{\xi_t^n \left(\widetilde{n}_t\right)^{\varphi}}{\lambda_t^U w_t} \tag{78}$$

$$\Theta_t^i = \tilde{\chi}_t^i \left(\hat{c}_t^i\right)^{-\sigma} \quad \forall \quad i = \{U, R, I\} \tag{79}$$

$$\tilde{\chi}_t^i = \left(\tilde{\chi}_{t-1}^i\right)^{1-v} \left(\hat{c}_t^i\right)^{\sigma v} \quad \forall \quad i = \{U, R, I\} \tag{80}$$

$$\begin{aligned} f_t^W = &\left(\frac{\epsilon_W - 1}{\epsilon_W}\right) \tilde{w}_t^{1-\epsilon_W} \widetilde{n}_t \\ &+ \left(\frac{\left(\omega_{UP}\beta^{UP} + (1-\omega_{UP})\beta^{RP}\right) + \beta_I}{2} \right) \theta_W \mathbb{E}_t \left\{ a_{t+1}^{-\sigma} \frac{\varrho_{t+1}\lambda_{t+1}^W}{\varrho_t \lambda_t^W} \frac{\pi_{t+1}^W}{\pi_{t+1}} \left(\frac{\pi_{t+1}^{\widetilde{W}}}{\pi_{t+1}^{I,W}} \right)^{\epsilon_W - 1} f_{t+1}^W \right\} \end{aligned} \tag{81}$$

$$\begin{aligned} f_t^W = &\tilde{w}_t^{-\epsilon_W(1+\varphi)} mc_t^W \widetilde{n}_t \\ &+ \left(\frac{\left(\omega_{UP}\beta^{UP} + (1-\omega_{UP})\beta^{RP}\right) + \beta_I}{2} \right) \theta_W \mathbb{E}_t \left\{ a_{t+1}^{-\sigma} \frac{\varrho_{t+1}\lambda_{t+1}^W}{\varrho_t \lambda_t^W} \frac{\pi_{t+1}^W}{\pi_{t+1}} \left(\frac{\pi_{t+1}^{\widetilde{W}}}{\pi_{t+1}^{I,W}} \right)^{\epsilon_W(1+\varphi)} f_{t+1}^W \right\} \end{aligned} \tag{82}$$

$$1 = (1-\theta_W)\tilde{w}_t^{1-\epsilon_W} + \theta_W \left(\frac{\pi_t^{I,W}}{\pi_t^W} \right)^{1-\epsilon_W} \tag{83}$$

$$\pi_t^{I,W} = a_{t-1}^{\alpha_W} a^{1-\alpha_W} \pi_{t-1}^{\kappa_W} \pi^{1-\kappa_W} \tag{84}$$

B.13 Fiscal Policy

$$\begin{aligned} \tau_t + R_{t-1}\frac{bs_{t-1}^G}{a_t \pi_t} + q_t^{BL} R_t^{BL} bl_{t-1}^G \frac{1}{a_t} + \chi s_t p_t^{Co\star} y_t^{Co} = &g_t + bs_t^G + q_t^{BL} bl_t^G + \gamma_D \frac{PD_t^D R_{t-1}^D d_{t-1}^F}{a_t \pi_t} \\ &+ \gamma_{BH} \frac{PD_t^H R_t^{BB} q_t^{BB} bb_{t-1}^{Pr}}{a_t} \end{aligned} \tag{85}$$

$$\tau_t = \alpha^T gdpn_t + \epsilon_t \left(bs^G - bs_t^G + q^{BL} bl^G - q_t^{BL} bl_t^G \right) \tag{86}$$

B.14 Monetary Policy and Rest of the World

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\alpha_R} \left[\left(\frac{(1-\alpha_E)\pi_t + \alpha_E \mathbb{E}_t\{\pi_{t+4}\}}{\pi_t^T} \right)^{\alpha_\pi} \left(\frac{gdp_t}{gdp_{t-1}} \right)^{\alpha_y} \right]^{1-\alpha_R} e_t^m \tag{87}$$

$$\frac{rer_t}{rer_{t-1}} = \frac{\pi_t^s \pi_t^\star}{\pi_t} \tag{88}$$

$$R_t^\star = R_t^W \exp\left\{ \frac{-\phi^\star}{100} \left(\frac{rer_t b_t^\star}{gdpn_t} - \frac{rerb^\star}{gdpn} \right) \right\} \xi_t^R z_t^\tau \tag{89}$$

$$x_t^{H\star} = \left(\frac{p_t^H}{rer_t}\right)^{-\eta^\star} y_t^\star \tag{90}$$

B.15 Aggregation and Market Clearing

$$y_t^C = c_t^P + c_t^I + i_t^K + i_t^H + g_t + \upsilon_t \tag{91}$$

$$c_t^P = \wp_U c_t^U + (1-\wp_U)\, c_t^R \tag{92}$$

$$\begin{aligned}\upsilon_t a_t \pi_t =& \gamma_D PD_t^D R_{t-1}^D d_{t-1}^F + \gamma_{BH} PD_t^H R_t^{BB} q_t^{BB} bb_{t-1}^{Pr} + \mu_e G_e\left(\bar{\omega}_t^e\right) R_t^e q_{t-1}^K k_{t-1} + \mu_I G_I\left(\bar{\omega}_t^I\right) R_t^H q_{t-1}^H h_{t-1}^I \\ &+ \mu_H G_H\left(\bar{\omega}_t^H\right) \tilde{R}_t^H l_{t-1}^H q_{t-1}^L + \mu_F G_F\left(\bar{\omega}_t^F\right) \tilde{R}_t^F l_{t-1}^F + \frac{\gamma_n}{2}\left(\frac{\widetilde{n}_t}{\widetilde{n}_{t-1}} - 1\right)^2 y_t^Z p_t^Z\end{aligned} \tag{93}$$

$$y_t^H = x_t^H + x_t^{H\star} \tag{94}$$

$$y_t^F = x_t^F \tag{95}$$

$$h_t = h_t^P + h_t^I \tag{96}$$

$$h_t^P = \wp_U h_t^U + (1-\wp_U)\, h_t^R \tag{97}$$

$$bl_t^{Pr} = \wp_U bl_t^U + (1-\wp_U)\, bl_t^R \tag{98}$$

$$bs_t^{Pr} = \wp_U bs_t^U \tag{99}$$

$$bb_t^{Tot} = \wp_U bb_t^U \tag{100}$$

$$b_t^{*Tot} = \wp_U b_t^{*U} \tag{101}$$

$$bl_t^{Pr} + bl_t^{CB} + bl_t^G = 0 \tag{102}$$

$$bs_t^{Pr} + bs_t^G = 0 \tag{103}$$

$$d_t^F = \wp_U d_t^U \tag{104}$$

$$\zeta_t^L = \left(\frac{q_t^{BL} bl_t^U + q_t^{BB} bb_t^U}{bs_t^U + rer_t b_t^{\star,U} + d_t^U}\right)^{\eta_\zeta} \epsilon_t^{L,S} \tag{105}$$

$$\tilde{R}_t^D = R_{t-1}^D \left(1 - \gamma_D PD_t^D\right) \tag{106}$$

$$\tilde{R}_t^{BB} = R_t^{BB} \left(1 - \gamma_{BH} PD_t^H\right) \tag{107}$$

$$R_t^{BL} = \frac{1}{q_t^{BL}} + \kappa_{BL} \tag{108}$$

$$R_t^{BB} = \frac{1}{q_t^{BB}} + \kappa_{BB} \tag{109}$$

$$R_t^{Nom,BL} = R_t^{BL} \pi_t \tag{110}$$

$$\frac{p_t^H}{p_{t-1}^H} = \frac{\pi_t^H}{\pi_t} \tag{111}$$

$$\frac{p_t^F}{p_{t-1}^F} = \frac{\pi_t^F}{\pi_t} \tag{112}$$

$$\pi_t^W = \frac{w_t}{w_{t-1}} a_t \pi_t \tag{113}$$

$$\pi_t^{\widetilde{W}} = \frac{\widetilde{w}_t}{\widetilde{w}_{t-1}} \pi_t^W \tag{114}$$

$$y_t^H \Xi_t^H = x_t^Z \tag{115}$$

$$y_t^Z = z_t \left(\frac{k_{t-1}}{a_t}\right)^\alpha \widetilde{n}_t^{1-\alpha} \tag{116}$$

$$y_t^Z = x_t^Z \tag{117}$$

$$\Xi_t^H = (1-\theta_H)\left(\tilde{p}_t^H\right)^{-\epsilon_H} + \theta_H \left(\frac{\pi_t^{I,H}}{\pi_t^H}\right)^{-\epsilon_H} \Xi_{t-1}^H \tag{118}$$

$$m_t = y_t^F \Xi_t^F \tag{119}$$

$$\Xi_t^F = (1-\theta_F)\left(\tilde{p}_t^F\right)^{-\epsilon_F} + \theta_F \left(\frac{\pi_t^{I,F}}{\pi_t^F}\right)^{-\epsilon_F} \Xi_{t-1}^F \tag{120}$$

$$n_t = \widetilde{n}_t \Xi_t^W \tag{121}$$

$$\Xi_t^W = (1-\theta_W)\tilde{w}_t^{-\epsilon_W} + \theta_W \left(\frac{\pi_t^{I,W}}{\pi_t^W}\right)^{-\epsilon_W} \Xi_{t-1}^W \tag{122}$$

$$n_t = n_t^P + n_t^I \tag{123}$$

$$n_t^P = n_t^I \tag{124}$$

$$n_t^P = \wp_U n_t^U + (1-\wp_U)\, n_t^R \tag{125}$$

$$n_t^U = n_t^R \tag{126}$$

$$gdp_t = c_t^P + c_t^I + i_t^K + i_t^H + g_t + x_t^{H\star} + y_t^{Co} - m_t \tag{127}$$
$$gdpn_t = c_t^P + c_t^I + i_t^K + i_t^H + g_t + tb_t \tag{128}$$
$$tb_t = p_t^H x_t^{H\star} + rer_t p_t^{Co\star} y_t^{Co} - rer_t \xi_t^m m_t \tag{129}$$
$$rer_t b_t^\star = \frac{rer_t}{a_t \pi_t^\star} b_{t-1}^\star R_{t-1}^\star + tb_t + rer_t ren^* - (1-\chi)\, rer_t p_t^{Co\star} y_t^{Co} \tag{130}$$

The exogenous processes are:

$$\log(z_t/z) = \rho_z \log(z_{t-1}/z) + u_t^z$$
$$\log(a_t/a) = \rho_a \log(a_{t-1}/a) + u_t^a$$
$$\log(\xi_t^n/\xi^n) = \rho_{\xi^n} \log(\xi_{t-1}^n/\xi^n) + u_t^{\xi^n}$$
$$\log(\xi_t^h/\xi^h) = \rho_{\xi^h} \log(\xi_{t-1}^h/\xi^h) + u_t^{\xi^h}$$
$$\log(\xi_t^i/\xi^i) = \rho_{\xi^i} \log(\xi_{t-1}^i/\xi^i) + u_t^{\xi^i}$$
$$\log(\xi_t^{ih}/\xi^{ih}) = \rho_{\xi^{ih}} \log(\xi_{t-1}^{ih}/\xi^{ih}) + u_t^{\xi^{ih}}$$
$$\log(\xi_t^R/\xi^R) = \rho_{\xi^R} \log(\xi_{t-1}^R/\xi^R) + u_t^{\xi^R}$$
$$\log(e_t^m/e^m) = \rho_{e^m} \log(e_{t-1}^m/e^m) + u_t^{e^m}$$
$$\log(g_t/g) = \rho_g \log(g_{t-1}/g) + u_t^g$$
$$\log(y_t^{Co}/y^{Co}) = \rho_{y^{Co}} \log(y_{t-1}^{Co}/y^{Co}) + u_t^{y^{Co}}$$
$$\log(\pi_t^\star/\pi^\star) = \rho_{\pi^\star} \log(\pi_{t-1}^\star/\pi^\star) + u_t^{\pi^\star}$$
$$\log(R_t^W/R^W) = \rho_{R^W} \log(R_{t-1}^W/R^W) + u_t^{R^W}$$
$$\log(y_t^\star/y^\star) = \rho_{y^\star} \log(y_{t-1}^\star/y^\star) + u_t^{y^\star}$$
$$\log(p_t^{Co\star}/p^{Co\star}) = \rho_{p^{Co\star}} \log(p_{t-1}^{Co\star}/p^{Co\star}) + u_t^{p^{Co\star}}$$
$$\log(\xi_t^m/\xi^m) = \rho_{\xi^m} \log(\xi_{t-1}^m/\xi^m) + u_t^{\xi^m}$$
$$\log(\sigma_t^I/\sigma^I) = \rho_{\sigma^I} \log(\sigma_{t-1}^I/\sigma^I) + u_t^{\sigma^I}$$
$$\log(\sigma_t^e/\sigma^e) = \rho_{\sigma^e} \log(\sigma_{t-1}^e/\sigma^e) + u_t^{\sigma^e}$$
$$\log(\sigma_t^F/\sigma^F) = \rho_{\sigma^F} \log(\sigma_{t-1}^F/\sigma^F) + u_t^{\sigma^F}$$
$$\log(\sigma_t^H/\sigma^H) = \rho_{\sigma^H} \log(\sigma_{t-1}^H/\sigma^H) + u_t^{\sigma^H}$$
$$\log(\epsilon_t^{L,S}/\epsilon^{L,S}) = \rho_{\epsilon^{L,S}} \log(\epsilon_{t-1}^{L,S}/\epsilon^{L,S}) + u_t^{\epsilon^{L,S}}$$
$$\log(bl_t^G/bl^G) = \rho_{bl^G} \log(bl_{t-1}^G/bl^G) + u_t^{bl^G}$$
$$\log(bl_t^{CB}/bl^{CB}) = \rho_{bl^{CB}} \log(bl_{t-1}^{CB}/bl^{CB}) + u_t^{bl^{CB}}$$
$$\log(\varrho_t/\varrho) = \rho_\varrho \log(\varrho_{t-1}/\varrho) + u_t^\varrho$$
$$\log(\xi_t^{\chi b}/\xi^{\chi b}) = \rho_\xi^{\chi b} \log(\xi_{t-1}^{\chi b}/\xi^{\chi b}) + u_t^{\xi^{\chi b}}$$
$$\log(\xi_t^{\chi e}/\xi^{\chi e}) = \rho_\xi^{\chi e} \log(\xi_{t-1}^{\chi e}/\xi^{\chi e}) + u_t^{\xi^{\chi e}}$$
$$\log(\xi_t^{roe}/\xi^{roe}) = \rho_\xi^{roe} \log(\xi_{t-1}^{roe}/\xi^{roe}) + u_t^{\xi^{roe}}$$
$$\log(z_t^\tau/z^\tau) = \rho_{z^\tau} \log(z_{t-1}^\tau/z^\tau) + u_t^{z^\tau}$$

with $u_t^j \sim \mathcal{N}(0,\, (\sigma^j)^2)$ for all j-exogenous variables defined above

C Steady State Computation

In this section we show how to compute the steady state for a given value of most of the parameters and all exogenous variables in the long run, except for:

$$R^W,\ \pi^\star,\ \sigma^F,\ \sigma^H,\ \sigma^e,\ \sigma^I,\ g,\ y^{Co},\ y^\star,\ o_{\hat{C}},\ ren^*,\ \xi^n.$$

that are determined endogenously by imposing values for the steady state of the following endogenous variables:

$$\pi^s,\ \xi^i=1,\ \xi^R,\ R^D,\ PD^F=PD^H,\ n,\ R^{nom,BL},\ R^{nom,I},\ R^L,\ p^H,\ r^{h,k}={}^{q^H h}/_{q^K k},\ s^g=g/gdpn,\ s^{Co}=p^{Co\star}y^{Co}rer/gdpn,$$
$$s^{tb}=tb/gdpn,\ s^{b*}=b^*rer/gdpn,\ \alpha_{BLG}=\frac{bl^G*q^{BL}}{gdpn},\ \alpha_{SG}=\frac{bs^G}{gdpn}$$

Start with (4), (5), (6), (87) (88) and (89):

$$R=\frac{\pi a^\sigma}{\beta_U};\quad \tilde{R}^D=R;\quad R^\star=\frac{R}{\pi^s};\quad \pi=\pi^T;\quad \pi^\star=\frac{\pi}{\pi^s};\quad R^W=\frac{R^\star}{\xi^R}$$

From (65), (74) and (111), (112):

$$\pi^{I,H}=\pi^{I,F}=\pi^H=\pi^F=\pi$$

From (84), (113) and (114) :

$$\pi^{I,W}=\pi^W=\pi^{\widetilde{W}}=a\pi$$

From (64), (73), (83), (62),(63), (71),(72), (81), (82), (118), (120) and (122):

$$\tilde{p}^H=\tilde{p}^F=\widetilde{w}=1$$
$$mc^H=\frac{\epsilon_H-1}{\epsilon_H}$$
$$mc^F=\frac{\epsilon_F-1}{\epsilon_F}$$
$$mc^W=\frac{\epsilon_W-1}{\epsilon_W}$$
$$\Xi^H=\Xi^F=\Xi^W=1$$

From (55) and (57):

$$q^K=1/\xi^i$$
$$q^H=\frac{a^{N_H\sigma}\varphi_0^H}{\beta_{UP}^{N_H}\xi^{ih}}\left(\frac{1-\left(\frac{\beta_{UP}\rho^{\varphi H}}{a^\sigma}\right)^{N_H+1}}{1-\frac{\beta_P\rho^{\varphi H}}{a^\sigma}}\right)$$

From (14) and (121):

$$R^H=\pi\left(1-\delta_H\right)$$
$$\widetilde{n}=n$$

From (35), (37), (38), (39) and (51):

$$\rho^H=\tilde{\rho}^H=\rho^F=\frac{a\pi}{1-\chi_b}$$

From (40), (106), R^D and using $PD^F=PD^H$

$$PD^D=\frac{1}{\gamma_D}\left(1-\frac{\tilde{R}^D}{R^D}\right)=PD^H=PD^F$$

From (12)

$$R^{BL}=\frac{R^{Nom,BL}}{\pi}$$
$$\beta_{RP}=\frac{a^\sigma}{R^{BL}}$$

From (17), (19) and (20)

$$R^I=\frac{R^{Nom,I}}{\pi}$$
$$\hat{R}^I=R^I$$
$$\hat{q}^L=\frac{1}{\hat{R}^I-\kappa_L}$$

$$q^L = \hat{q}^L$$

From (7) and (8)

$$\tilde{R}^{BB} = R^{BL}$$

From (107)

$$R^{BB} = \frac{\tilde{R}^{BB}}{1 - \gamma_D PD^H}$$

From (109)

$$q^{BB} = \frac{1}{R^{BB} - \kappa_{BB}}$$

From (108)

$$q^{BL} = \frac{1}{R^{BL} - \kappa_B}$$
$$\Delta l = a$$

Numerical solution for $\bar{\omega}^F$ and σ^F using (42), (44) and (46)

$$\bar{\omega}^F - \left[1 - \Gamma_F\left(\bar{\omega}^F, \sigma^F\right)\right]\left(\frac{1-\phi_F}{\phi_F}\right)\frac{R^D}{\tilde{\rho}^F} = 0$$
$$PD^F - F_F\left(\bar{\omega}^F, \sigma^F\right) = 0$$

Numerical solution for $\bar{\omega}^H$ and σ^H using (48), (50) and (53)

$$\bar{\omega}^H - \left[1 - \Gamma_H\left(\bar{\omega}^H, \sigma^H\right)\right]\left(\frac{1-\phi_H}{\phi_H}\right)\frac{R^{BB}}{\rho^H}\pi = 0$$
$$PD^H - F_H\left(\bar{\omega}^H, \sigma^H\right) = 0$$

Then, from (44) and (50):

$$\tilde{R}^F = \frac{\phi_F \rho^F}{1 - \Gamma_F\left(\bar{\omega}^F, \sigma^F\right)}$$
$$\tilde{R}^H = \frac{\phi_H \rho^H}{1 - \Gamma_H\left(\bar{\omega}^H, \sigma^H\right)}$$

Numerical solution for $\bar{\omega}^e$ and σ^e: Use (33) in (32), then use (44), (45), (26) and (31). Later combine (28) and (45) to obtain

$$\frac{\Gamma_e'\left(\bar{\omega}^e, \sigma^e\right) - \mu_e G_e'\left(\bar{\omega}^e, \sigma^e\right)}{\Gamma_e'\left(\bar{\omega}^e, \sigma^e\right)} - \frac{\left(1-\chi_e\right)\tilde{R}^F}{a\pi} = 0$$
$$R^L - \frac{\tilde{R}^F \bar{\omega}^e}{\Gamma_e\left(\bar{\omega}^e, \sigma^e\right) - \mu_e G_e\left(\bar{\omega}^e, \sigma^e\right)} = 0$$

From (34):

$$PD^e = F_e\left(\bar{\omega}^e\right)$$

Numerical solution for $\bar{\omega}^I$ and σ^I: use (50) and (24) in (22). Also, use (52) in (18)

$$\frac{\Gamma_I'\left(\bar{\omega}^I, \sigma^I\right) - \mu_I G_I'\left(\bar{\omega}^I, \sigma^I\right)}{\Gamma_I'\left(\bar{\omega}^I, \sigma^I\right)} - \frac{\beta_I \tilde{R}^H}{a^\sigma \pi} = 0$$
$$R^I - \frac{\tilde{R}^H \bar{\omega}^I}{\pi\left[\Gamma_I\left(\bar{\omega}^I, \sigma^I\right) - \mu_I G_I\left(\bar{\omega}^I, \sigma^I\right)\right]} = 0$$

From (25):

$$PD^I = F_I\left(\bar{\omega}^I\right)$$

From (30), (26), (31) and (45):

$$R^e = \frac{\tilde{R}^F a\pi}{a\pi\left[\Gamma_e\left(\bar{\omega}^e\right) - \mu_e G_e\left(\bar{\omega}^e\right)\right] + \left[1 - \Gamma_e\left(\bar{\omega}^e\right)\right]\left(1-\chi_e\right)\tilde{R}^F}$$

From (27):

$$r^K = q^K\left[\frac{R^e}{\pi} - \left(1 - \delta_K\right)\right]$$

From (66) and (69):

$$p^Z = p^H mc^H$$
$$mc^Z = p^Z$$

From (67), (68), (116), (117) and (54) :

$$w = \left[\frac{\alpha^\alpha (1-\alpha)^{1-\alpha} mc^Z z}{(r^k)^\alpha}\right]^{\frac{1}{1-\alpha}}$$
$$k = \frac{\alpha}{1-\alpha}\tilde{n}\frac{w}{r^k}a$$
$$y^Z = z\left(\frac{k}{a}\right)^\alpha \tilde{n}^{1-\alpha}$$
$$x^Z = y^Z$$
$$i = k\left[\frac{1-(1-\delta_K)/a}{\xi^i}\right]$$

Also, from (115)

$$y^H = \frac{x^Z}{\Xi^H}$$

From (26), (29), (30), (31) and (33):

$$\psi^e = [1-\Gamma_e(\bar{\omega}^e)]\frac{R^e q^K k}{a\pi}$$
$$n^e = (1-\chi_e \xi^{\chi_e})\psi^e$$
$$c^e = \chi_e \xi^{\chi_e}\psi^e$$
$$\lambda^e = \frac{\Gamma^{e'}(\bar{\omega}^e)}{(1-\Gamma^F(\bar{\omega}^F))\left[\Gamma^{e'}(\bar{\omega}^e) - \mu^e G^{e'}(\bar{\omega}^e)\right]}$$
$$l^F = q^K k - n^e$$

From (43), (41) and (104):

$$e^F = \phi_F l^F$$
$$d^F = l^F - e^F$$
$$d^U = d^F/\wp_U$$

From $r^{h,k} = {}^{q^H h}/{}_{q^K k}$, (56) to (58):

$$h = \frac{r^{h,k} q^K k}{q^H}$$
$$i^{AH} = \frac{h a^{N_H}}{\xi^{ih}}\left[1-\left(\frac{1-\delta_H}{a}\right)\right]$$
$$i^H = i^{AH}\varphi_0^H\left[\frac{1-\left(\frac{\rho^{\varphi H}}{a}\right)^{N_H+1}}{1-\frac{\rho^{\varphi H}}{a}}\right]$$

From (59), (60) and (61):

$$p^F = \left[\frac{1-\omega(p^H)^{1-\eta}}{1-\omega}\right]^{\frac{1}{1-\eta}}$$

From (70):

$$rer = mc^F p^F/\xi^m$$

Numerical solution for l_h iterating over the following equation up until $\Delta^l \approx 0$ (see Appendix C.1)

$$\Delta^l = gdpn - (c^P + c^I + i + i^H + s^g gdpn + s^{tb} gdpn)$$

From (18):

$$h^I = \frac{R^I q^L l^H}{\bar{\omega}^I R^H q^H}$$

from (49):

$$e^H = \phi_H q^L l^H$$

From (36), (37), (39) and (47):

$$n^b = e^F + e^H$$

$$\psi^b = \frac{n^b}{1-\chi_b \xi^{\chi_b}}$$
$$c^b = \chi_b \xi^{\chi_b} \psi^b$$
$$bb^{Tot} = (1-\phi_H)\frac{q^L l^H}{q^{BB}}$$

Then, from (93):

$$\upsilon = \frac{1}{a\pi}\left(\begin{array}{c} \gamma_D PD^D R^D d^F + +\gamma_{BB} PD^H R^{BB} q^{BB} bb^{Tot} + \mu_e G_e(\bar{\omega}^e) R^e q^K k \\ +\mu_I G_I(\bar{\omega}^I) R^H q^H h^I + \mu_H G_H(\bar{\omega}^H)\tilde{R}^H q^L l^H + \mu_F G_F(\bar{\omega}^F)\tilde{R}^F l^F \end{array}\right)$$

From (128), (91), (129), (59), (60), (61), (119), (94) and (95):

$$gdpn = \frac{p^H y^H + (p^F)^{-\eta}(p^F - rer\xi^m \Xi^F)(1-\omega)\upsilon - \upsilon}{1 - s^{Co} - (1-s^{tb})(p^F)^{-\eta}(p^F - rer\xi^m\Xi^F)(1-\omega)}$$

From their definitions:

$$tb = s^{tb} gdpn$$
$$g = s^g gdpn$$
$$y^{Co} = \frac{s^{Co} gdpn}{p^{Co\star} rer}$$
$$b^{*Tot} = \frac{s^{b*} gdpn}{rer}$$

From (60), (61),(90), (91), (94), (95) , (119) and (128):

$$y^C = gdpn + \upsilon - tb$$
$$x^F = (1-\omega)(p^F)^{-\eta} y^C$$
$$x^H = \omega(p^H)^{-\eta} y^C$$
$$x^{H\star} = y^H - x^H$$
$$y^\star = x^{H\star}\left(\frac{p^H}{rer}\right)^{\eta\star}$$
$$y^F = x^F$$
$$m = y^F \Xi^F$$

From (96):

$$h^P = h - h^I$$

From (23):

$$c^I = \frac{wn}{2} + q^H h^I\left[(1-\Gamma_I)\frac{R^H}{a\pi} - 1\right] + q^L l^H$$

From (21) and (16):

$$o_{\hat{C}} = \left\{(a)^{-\sigma\eta_{\hat{C}}}(\xi^h)^{\eta_{\hat{C}}-1}\left(\frac{ac^I\left(1-\frac{\phi_c}{a}\right)}{h^I\left(1-\frac{\phi_{hh}}{a}\right)}\right)\left(\frac{1}{\beta_I}\left[q^H - (\Gamma_I - \mu_I G_I)\frac{R^H q^H}{\tilde{R}^H}\right] - a^{-\sigma}(1-\Gamma_I)\frac{R^H}{\pi}q^H\right)^{-\eta_{\hat{C}}} + 1\right\}^{-1}$$

Then from (15) we can compute

$$\hat{c}^I = \left[(1-o_{\hat{C}})^{\frac{1}{\eta_{\hat{C}}}}\left(c^I\left(1-\frac{\phi_c}{a}\right)\right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}} + (o_{\hat{C}})^{\frac{1}{\eta_{\hat{C}}}}\left(\frac{\xi^h h^I}{a}\left(1-\frac{\phi_{hh}}{a}\right)\right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}}\right]^{\frac{\eta_{\hat{C}}}{\eta_{\hat{C}}-1}}$$

From (16):

$$\lambda^I = \left\{\left(\hat{c}^I\right)^{-\sigma}\right\}\left(\frac{(1-o_{\hat{C}})\hat{c}^I}{c^I\left(1-\frac{\phi_c}{a}\right)}\right)^{\frac{1}{\eta_{\hat{C}}}}$$

From (21) and (22)

$$\lambda^H = \frac{\lambda^I}{\rho^H \phi_H}$$

Use ratios $\alpha_{BLG} = \frac{bl^G q^{BL}}{gdpn}$ and $\alpha_{SG} = \frac{bs^G}{gdpn}$

$$bl^G = \alpha_{BLG}\frac{gdpn}{q^{BL}}$$
$$bs^G = \alpha_{BSG} gdpn$$

Then from (102) and (103), and normalizing $bl^{CB}=1$

$$bl^{Pr} = -bl^{G}$$
$$bs^{Pr} = -bs^{G}$$

We can solve for bond holdings of the unrestricted households Also, from (99), (100) and (101)

$$bs^{U} = \frac{bs^{Pr}}{\wp^{U}}$$
$$b^{*U} = \frac{b^{*Tot}}{\wp^{U}}$$
$$bb^{U} = \frac{bb^{tot}}{\wp^{U}}$$

Then using the (exogenously given) ratio of long to short term instruments held by the unrestricted patient household, ω_{BL}

$$bl^{u} = \frac{\omega_{BL} * (bs^{u} + rer * b^{*U} + d^{U}) - bb^{U} q^{BB}}{q_{BL}}$$

We can then, using (98) results in long term bonds held by the restricted household of

$$bl^{R} = \frac{bl^{Pr} - \wp_{U} bl^{U}}{1-\wp_{U}}$$

From 102

$$bl^{CB} = 1$$

Next, we solve for h^R, c^R, $\hat{c}^R$, λ^R. From (10) and (11) and the restricted household budget constraint (13)

$$h^{R} = \frac{q^{BL} bl^{R} \left(\frac{R^{BL}}{a} - 1\right) + \frac{wn}{2}}{q^{H} - \frac{q^{H}}{a}(1-\delta_{H}) + aux_{1}}$$

with aux_1

$$aux_{1} = (a)^{\sigma\eta_{\hat{C}}-1} (\xi^{h})^{1-\eta_{\hat{C}}} \left(\frac{q^{H}}{\beta_{P}} - (1-\delta_{H})\, a^{-\sigma} q^{H}\right)^{\eta_{\hat{C}}} \frac{(1-o_{\hat{C}})\left(1-\frac{\phi_{hh}}{a}\right)}{o_{\hat{C}}\left(1-\frac{\phi_{c}}{a}\right)}$$

and

$$c^{R} = h^{R} aux_{1}$$

From (9):

$$\hat{c}^{R} = \left[(1-o_{\hat{C}})^{\frac{1}{\eta_{\hat{C}}}} \left(c^{R}\left(1-\frac{\phi_{c}}{a}\right)\right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}} + (o_{\hat{C}})^{\frac{1}{\eta_{\hat{C}}}} \left(\xi^{h} \frac{h^{R}}{a}\left(1-\frac{\phi_{hh}}{a}\right)\right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}}\right]^{\frac{\eta_{\hat{C}}}{\eta_{\hat{C}}-1}}$$

From (10):

$$\lambda^{R} = \left\{\left(\hat{c}^{R}\right)^{-\sigma}\right\} \left(\frac{(1-o_{\hat{C}})\,\hat{c}^{R}}{c^{R}\left(1-\frac{\phi_{c}}{a}\right)}\right)^{\frac{1}{\eta_{\hat{C}}}}$$

Also, from (97) we get

$$h^{U} = \frac{h^{P} - (1-\wp_{U}) h^{R}}{\wp_{U}}$$

which together with (2) and (3) lets us solve for c^U

$$c^{U} = h^{U} (a)^{\sigma\eta_{\hat{C}}-1} (\xi^{h})^{1-\eta_{\hat{C}}} \left(\frac{q^{H}}{\beta_{P}} - (1-\delta_{H})\, a^{-\sigma} q^{H}\right)^{\eta_{\hat{C}}} \frac{(1-o_{\hat{C}})\left(1-\frac{\phi_{hh}}{a}\right)}{o_{\hat{C}}\left(1-\frac{\phi_{c}}{a}\right)}$$

From (1) we solve for $\hat{c}^U$

$$\hat{c}^{U} = \left[(1-o_{\hat{C}})^{\frac{1}{\eta_{\hat{C}}}} \left(c^{U}\left(1-\frac{\phi_{c}}{a}\right)\right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}} + (o_{\hat{C}})^{\frac{1}{\eta_{\hat{C}}}} \left(\xi^{h} \frac{h^{U}}{a}\left(1-\frac{\phi_{hh}}{a}\right)\right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}}\right]^{\frac{\eta_{\hat{C}}}{\eta_{\hat{C}}-1}}$$

and from (2) we obtain λ^U

$$\lambda^{U} = \left(\hat{c}^{U}\right)^{-\sigma} \left(\frac{(1-o_{\hat{C}})\,\hat{c}^{U}}{c^{U}\left(1-\frac{\phi_{c}}{a}\right)}\right)^{\frac{1}{\eta_{\hat{C}}}}$$

From (76):

$$\lambda^P = \wp_U \lambda^U + (1 - \wp_U)\,\lambda^R$$

From (92):

$$c^P = \wp_U c^U + (1 - \wp_U)\,c^R$$
$$c = c^p + c^i$$

From (123), (124), (125), (126)

$$n^P = \frac{n}{2} = n^I = n^U = n^R$$

From (79), (80) and (77):

$$\tilde{\chi}^U = \left(\hat{c}^U\right)^\sigma$$
$$\Theta^U = 1$$
$$\tilde{\chi}^I = \left(\hat{c}^I\right)^\sigma$$
$$\Theta^I = 1$$
$$\tilde{\chi}^R = \left(\hat{c}^R\right)^\sigma$$
$$\Theta^R = \tilde{\chi}^R \left(\hat{c}^R\right)^{-\sigma}$$

$$\Theta = \frac{\left(\wp_U \Theta^U + (1 - \wp_U)\,\Theta^R\right) + \Theta^I}{2} = 1$$

From (75) and (78):

$$\lambda^W = \frac{\lambda^P + \lambda^I}{2}, \qquad \xi^n = \frac{mc^W \lambda^W w}{\Theta \widetilde{n}^\varphi}$$

From (127) and (130):

$$gdp = c + i + i^h + g + x^{H\star} + y^{Co} - m$$
$$ren^* = b^\star \left(1 - \frac{R^\star}{a\pi^\star}\right) - \frac{tb}{rer} + (1 - \chi)\,p^{Co\star} y^{Co}$$

From (7) and (105)

$$\epsilon^{L,S} = \beta_U R^{BL} a^{-\sigma} - 1$$

From (105) :

$$\zeta^L = \epsilon^{L,S}$$

From (85):

$$\tau = g + dia - bs^G \left(\frac{R}{a\pi} - 1\right) - q^{BL} bl^G \left(\frac{R^{BL}}{a} - 1\right) - \chi rer p^{Co\star} y^{Co}$$

From (86):

$$\alpha^T = \frac{\tau}{gdpn}$$

Finally, from (63), (72) and (82):

$$f^H = \frac{(\tilde{p}^H)^{-\epsilon_H} y^H mc^H}{1 - \beta_{UP}\theta_H a^{1-\sigma}}, \quad f^F = \frac{(\tilde{p}^F)^{-\epsilon_F} y^F mc^F}{1 - \beta_{UP}\theta_F a^{1-\sigma}}, \quad f^W = \frac{\tilde{w}^{-\epsilon_W(1+\varphi)} mc^W \widetilde{n}}{1 - \left(\frac{(\omega_{UP}\beta^{UP} + (1-\omega_{UP})\beta^{RP}) + \beta_I}{2}\right)\theta_W a^{1-\sigma}}$$

C.1 Numerical solution for l^H

First, guess l^H. Then, from (18) solve for h^I:

$$h^I = \frac{R^I q^L l^H}{\bar{\omega}^I R^H q^H}$$

From (49) and (47):

$$bb^{Tot} = (1-\phi_H)\frac{q^L l^H}{q^{BB}}$$

Then, from (93):

$$\upsilon = \frac{1}{a\pi}\begin{pmatrix} \gamma_D PD^D R^D d^F + +\gamma_{BB} PD^H R^{BB} q^{BB} bb^{Tot} + \mu_e G_e(\bar{\omega}^e) R^e q^K k \\ +\mu_I G_I(\bar{\omega}^I) R^H q^H h^I + \mu_H G_H(\bar{\omega}^H)\tilde{R}^H q^L l^H + \mu_F G_F(\bar{\omega}^F)\tilde{R}^F l^F \end{pmatrix}$$

From (128), (91), (129), (59), (60), (61), (119), (94) and (95):

$$gdpn = \frac{p^H y^H + (p^F)^{-\eta}(p^F - rer\xi^m \Xi^F)(1-\omega)\upsilon - \upsilon}{1 - s^{Co} - (1-s^{tb})(p^F)^{-\eta}(p^F - rer\xi^m\Xi^F)(1-\omega)}$$

From (96):

$$h^P = h - h^I$$

From (23):

From (21) and (16):

$$o_{\hat{C}} = \left\{(a)^{-\sigma\eta_{\hat{C}}}(\xi^h)^{\eta_{\hat{C}}-1}\left(\frac{ac^I\left(1-\frac{\phi_c}{a}\right)}{h^I\left(1-\frac{\phi_{hh}}{a}\right)}\right)\left(\frac{1}{\beta_I}\left[q^H - (\Gamma_I - \mu_I G_I)\frac{R^H q^H}{\tilde{R}^H}\right] - a^{-\sigma}(1-\Gamma_I)\frac{R^H}{\pi}q^H\right)^{-\eta_{\hat{C}}} + 1\right\}^{-1}$$

Use ratios $\alpha_{BLG} = \frac{bl^G}{gdpn q^{BL}}$ and $\alpha_{SG} = \frac{bs^G}{gdpn}$

$$bl^G = \alpha_{BLG}\frac{gdpn}{q^{BL}}$$
$$bs^G = \alpha_{BSG}gdpn$$

Then from (102) and (103), and normalizing $bl^{CB} = 0$

$$bl^{Pr} = -bl^G$$
$$bs^{Pr} = -bs^G$$

Also, from (99) and (100)

$$bs^U = \frac{bs^{Pr}}{\wp^U}, \qquad bb^U = \frac{bb^{tot}}{\wp^U}$$

Use ratio $s^{b*} = b^* rer/gdpn$, and (101)

$$b^{*Tot} = s^{b*} * gdpn/rer, \qquad b^{*U} = \frac{b^{*Tot}}{\wp^U}$$

Then using the ratio of long to short term instruments held by the unrestricted patient household, ω_{BL}

$$bl^u = \frac{\omega_{BL} * (bs^u + rer * b^{*U} + d^U) - bb^U q^{BB}}{q_{BL}}$$

which using (98) results in long term bonds held by the restricted household of

$$bl^R = \frac{bl^{Pr} - \wp_U bl^U}{1-\wp_U}$$

From (10) and (11) and the restricted household budget constraint (13)

$$h^R = \frac{q^{BL} bl^R\left(\frac{R^{BL}}{a} - 1\right) + \frac{wn}{2}}{q^H - \frac{q^H}{a}(1-\delta_H) + aux_1}$$

with aux_1

$$aux_1 = (a)^{\sigma\eta_{\hat{C}}-1}(\xi^h)^{1-\eta_{\hat{C}}}\left(\frac{q^H}{\beta_P} - (1-\delta_H)a^{-\sigma}q^H\right)^{\eta_{\hat{C}}}\frac{(1-o_{\hat{C}})\left(1-\frac{\phi_{hh}}{a}\right)}{o_{\hat{C}}\left(1-\frac{\phi_c}{a}\right)}$$

and

$$c^R = h^R aux_1$$

Also, from (97) we get

$$h^U = \frac{h^P - (1-\wp_U)h^R}{\wp_U}$$

which together with (2) and (3) lets us solve for c^U

$$c^U = h^U (a)^{\sigma \eta_{\hat{C}} - 1} (\xi^h)^{1-\eta_{\hat{C}}} \left(\frac{q^H}{\beta_P} - (1-\delta_H)\, a^{-\sigma} q^H \right)^{\eta_{\hat{C}}} \frac{(1-o_{\hat{C}}) \left(1 - \frac{\phi_{hh}}{a}\right)}{o_{\hat{C}} \left(1 - \frac{\phi_c}{a}\right)}$$

From (92):

$$c^P = \wp_U c^U + (1-\wp_U)\, c^R$$

Then, the following equation must hold:

$$gdpn = c^P + c^I + i + i^H + s^g gdpn + s^{tb} gdpn$$

If it does not, update guess of l^H and repeat.

D Steady state for capital requirements comparative statics

For a given value of capital requirements ϕ_f, ϕ_h we use estimated and calibrated parameters: related to real sector α, α^{BSG}, α^{BLG}, β_U, β_R, β_I, δ_K, δ_K, ϵ_F, ϵ_H, ϵ_W, N_H, κ, κ_{BL}, κ_{BB}, σ, χ, ω, ω_U, ω_{BL}, η, η^*, $\eta_{\hat{C}}$, θ_F, θ_H, θ_W, η_{ζ_L}; financial sector : χ_b, χ_e, γ_d, γ_{bh}, μ_e, μ_f, μ_h, μ_i, σ^e, σ^F, σ^H, σ^I, ξ^{χ_e}, ξ^{χ_b}; preference parameters and external sector parameters: $O_{\hat{C}}$, ϕ_c, ϕ_{hh}, $\rho^{\varphi H}$, φ, φ_0^H, a, bl^{cb}, $\epsilon^{L,S}$, g, n, $r^{h,k}$, π^T, p^{Co}, π^*, R^W, ξ^h, ξ^i, ξ^{ih}, ξ^m, ξ^n, ξ^R, y^*, y^{Co}, z, bl^G, bs^G, b^{*Tot} to compute the steady state of the model consistent with capital requirements different from that of the 2001-2019 period

Consider ϕ^F and ϕ^H total capital requirements including regulatory minimum capital, voluntary buffers and the neutral level (if any) for the CCyB requirement.

$$\phi^F = (\phi^F_{Reg} + \phi^F_{Vol} + CCyB)$$
$$\phi^H = 0.6(\phi^H_{Reg} + \phi^H_{Vol} + CCyB)$$

Use (4), (5), (6), (87) (88) and (89):

$$\pi = \pi^T; \quad R = \frac{\pi a^\sigma}{\beta_U}; \quad \tilde{R}^D = R; \quad \pi^s = \frac{\pi}{\pi^\star}; \quad R^\star = \frac{R}{\pi^s}; \quad R^W = \frac{R^\star}{\xi^R}$$

From (65), (74) and (111), (112):

$$\pi^H = \pi^F = \pi^{I,H} = \pi^{I,F} = \pi$$

From (84), (113) and (114):

$$\pi^W = \pi^{\widetilde{W}} = \pi^{I,W} = a\pi$$

From (62),(63),(64), (71),(72),(73), (81), (82), (83), (118), (120) and (122):

$$\tilde{p}^H = \tilde{p}^F = \widetilde{w} = 1$$
$$mc^H = \frac{\epsilon_H - 1}{\epsilon_H}$$
$$mc^F = \frac{\epsilon_F - 1}{\epsilon_F}$$
$$mc^W = \frac{\epsilon_W - 1}{\epsilon_W}$$
$$\Xi^H = \Xi^F = \Xi^W = 1$$

From (55) and (57):

$$q^K = 1/\xi^i; \quad \nabla l = a$$
$$q^H = \frac{a^{N_H \sigma}\varphi_0^H}{\beta_{UP}^{N_H}\xi^{ih}} \left(\frac{1 - \left(\frac{\beta_{UP}\rho^{\varphi H}}{a^\sigma}\right)^{N_H+1}}{1 - \frac{\beta_P \rho^{\varphi H}}{a^\sigma}} \right)$$

From (14) and (121):

$$R^H = \pi(1 - \delta_H)$$
$$\widetilde{n} = n$$

From (35), (37), (38), (39) and (51):

$$\rho^H = \tilde{\rho}^H = \rho^F = \frac{a\pi}{1 - \chi_b}$$

From (12) and (110)

$$R^{BL} = \frac{a^\sigma}{\beta_{RP}}$$
$$R^{Nom,BL} = R^{BL}\pi$$

From (7) and (8)

$$\tilde{R}^{BB} = R^{BL}$$

From (108)

$$q^{BL} = \frac{1}{R^{BL} - \kappa_B}$$

Given σ^F and the previous result for $\tilde{R}^D$, use a numerical solution for $\bar{\omega}^F$ and R^D using (42), (44) and (106)

$$\bar{\omega}^F - \left[1 - \Gamma_F\left(\bar{\omega}^F, \sigma^F\right)\right]\left(\frac{1-\phi_F}{\phi_F}\right)\frac{R^D}{\tilde{\rho}^F} = 0$$
$$PD^F - \frac{1}{\gamma_D}\left(1 - \frac{\tilde{R}^D}{R^D}\right) = 0$$

And, from (44)

$$\tilde{R}^F = \frac{\phi_F \rho^F}{1-\Gamma_F\left(\bar{\omega}^F,\sigma^F\right)}$$

Next, given σ^H and previous results for $\tilde{R}^{BB}$, use (48), (50) and (107) to find $\bar{\omega}^H$ and R^{BB} numerically,

$$\bar{\omega}^H - \left[1-\Gamma_H\left(\bar{\omega}^H,\sigma^H\right)\right]\left(\frac{1-\phi_H}{\phi_H}\right)\frac{R^{BB}}{\rho^H}\pi = 0$$

$$\tilde{R}^{BB} = R^{BB}\left(1-\gamma_{BH}PD^H\right)$$

Then, from (48), (53) and (109):

$$\tilde{R}^H = \frac{\phi_H \rho^H}{1-\Gamma_H\left(\bar{\omega}^H,\sigma^H\right)}$$

$$PD^H = F_H\left(\bar{\omega}^H,\sigma^H\right)$$

$$q^{BB} = \frac{1}{R^{BB}-\kappa_{BB}}$$

Use (33) in (32), then use (44), (45), (26) and (31) to solve for $\bar{\omega}^e$

$$\frac{\Gamma_e'\left(\bar{\omega}^e,\sigma^e\right)-\mu_e G_e'\left(\bar{\omega}^e,\sigma^e\right)}{\Gamma_e'\left(\bar{\omega}^e,\sigma^e\right)} - \frac{\left(1-\chi_e\right)\tilde{R}^F}{a\pi} = 0$$

Then, from (34):

$$PD^e = F_e\left(\bar{\omega}^e\right)$$

Combine (28) and (45) to obtain

$$R^L = \frac{\tilde{R}^F\bar{\omega}^e}{\Gamma_e\left(\bar{\omega}^e,\sigma^e\right)-\mu_e G_e\left(\bar{\omega}^e,\sigma^e\right)}$$

Go back to (33) in (32) to obtain

$$\lambda^e = \frac{\Gamma^{e'}\left(\bar{\omega}^e\right)}{\left(1-\Gamma^F\left(\bar{\omega}^F\right)\right)\left[\Gamma^{e'}\left(\bar{\omega}^e\right)-\mu^e G^{e'}\left(\bar{\omega}^e\right)\right]}$$

$$R^e = \left\{\frac{\left[1-\Gamma_e\left(\bar{\omega}^e\right)\right]}{\lambda^e} + \left[1-\Gamma_F(\bar{\omega}^F)\right]\left[\Gamma_e\left(\bar{\omega}^e\right)-\mu_e G_e\left(\bar{\omega}^e\right)\right]\right\}^{-1}\rho^F\phi_F$$

From (27):

$$r^K = q^K\left[\frac{R^e}{\pi}-\left(1-\delta_K\right)\right]$$

Numerical solution for $\bar{\omega}^I$ using (50) and (22)

$$\frac{\Gamma_I'\left(\bar{\omega}^I,\sigma^I\right)-\mu_I G_I'\left(\bar{\omega}^I,\sigma^I\right)}{\Gamma_I'\left(\bar{\omega}^I,\sigma^I\right)} - \frac{\beta_I\tilde{R}^H}{a^\sigma\pi} = 0$$

From (25):

$$PD^I = F_I\left(\bar{\omega}^I\right)$$

From (18) and (52)

$$\hat{R}^I = \frac{\tilde{R}^H\bar{\omega}^I}{\pi\left[\Gamma_I\left(\bar{\omega}^I,\sigma^I\right)-\mu_I G_I\left(\bar{\omega}^I,\sigma^I\right)\right]}$$

and from (17), (19) and (20)

$$\hat{q}^L = \frac{1}{\hat{R}^I-\kappa_L}$$

$$q^L = \hat{q}^L$$

$$\hat{R}^I = R^I$$

From (20)

$$R^{Nom,I} = R^I\pi$$

Using the normalization $p^H = 1$, and from (66) and (69):

$$p^Z = p^H mc^H$$

$$mc^Z = p^Z$$

From (67), (68), (116), (117) and (54) :

$$w = \left[\frac{\alpha^\alpha (1-\alpha)^{1-\alpha} mc^Z z}{(r^k)^\alpha} \right]^{\frac{1}{1-\alpha}}$$
$$k = \frac{\alpha}{1-\alpha} \tilde{n} \frac{w}{r^k} a$$
$$y^Z = z \left(\frac{k}{a} \right)^\alpha \tilde{n}^{1-\alpha}$$
$$x^Z = y^Z$$
$$i = k \left[\frac{1-(1-\delta_K)/a}{\xi^i} \right]$$

Also, from (115)

$$y^H = \frac{x^Z}{\Xi^H}$$

From (26), (29), (30), (31) and (33):

$$\psi^e = [1 - \Gamma_e(\bar{\omega}^e)] \frac{R^e q^K k}{a\pi}$$
$$n^e = (1 - \chi_e \xi^{\chi_e}) \psi^e$$
$$c^e = \chi_e \xi^{\chi_e} \psi^e$$
$$l^F = q^K k - n^e$$

From (43), (41) and (104):

$$e^F = \phi_F l^F$$
$$d^F = l^F - e^F$$
$$d^U = d^F / \wp_U$$

From (59), (60) and (61):

$$p^F = \left[\frac{1-\omega (p^H)^{1-\eta}}{1-\omega} \right]^{\frac{1}{1-\eta}}$$

From (70):

$$rer = mc^F p^F / \xi^m$$

Next, we can find l^H, h^I, c^I solving the three equation system by (18), (23) and (21)

$$h^I = \frac{R^I q^L l^H}{\bar{\omega}^I R^H q^H}$$
$$c^I = \frac{wn}{2} + q^H h^I \left[(1-\Gamma_I) \frac{R^H}{a\pi} - 1 \right] + q^L l^H$$
$$\Delta^l = o_{\hat{C}} - \left\{ (a)^{-\sigma \eta_{\hat{C}}} (\xi^h)^{\eta_{\hat{C}}-1} \left(\frac{a c^I \left(1 - \frac{\phi_c}{a}\right)}{h^I \left(1 - \frac{\phi_{hh}}{a}\right)} \right) \left(\frac{1}{\beta_I} \left[q^H - (\Gamma_I - \mu_I G_I) \frac{R^H q^H}{\tilde{R}^H} \right] - a^{-\sigma} (1-\Gamma_I) \frac{R^H}{\pi} q^H \right)^{-\eta_{\hat{C}}} + 1 \right\}^{-1}$$

Then from (15) we can compute

$$\hat{c}^I = \left[(1 - o_{\hat{C}})^{\frac{1}{\eta_{\hat{C}}}} \left(c^I \left(1 - \frac{\phi_c}{a}\right) \right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}} + (o_{\hat{C}})^{\frac{1}{\eta_{\hat{C}}}} \left(\frac{\xi^h h^I}{a} \left(1 - \frac{\phi_{hh}}{a}\right) \right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}} \right]^{\frac{\eta_{\hat{C}}}{\eta_{\hat{C}}-1}}$$

and from (16) and (24), respectively:

$$\lambda^I = \left\{ \left(\hat{c}^I\right)^{-\sigma} \right\} \left(\frac{(1 - o_{\hat{C}}) \hat{c}^I}{c^I \left(1 - \frac{\phi_c}{a}\right)} \right)^{\frac{1}{\eta_{\hat{C}}}} ; \quad \lambda^H = \frac{\lambda^I}{\rho^H \phi^H}$$

Also, from (49):

$$e^H = \phi_H q^L l^H$$

From (39), (37), (36), and (47):

$$n^b = e^F + e^H$$
$$\psi^b = \frac{n^b}{1-\chi_b\xi^{\chi_b}}$$
$$c^b = \chi_b\xi^{\chi_b}\psi^b$$
$$bb^{Tot} = \frac{q^L l^H - e^H}{q^{BB}}$$

From (40)

$$PD^D = \frac{q^{BB}bb^{Tot}PD^H + d^F PD^F}{q^{BB}bb^{Tot} + d^F}$$

From (90), (94), (61), (60) (95) and (119)

$$x^{H\star} = \frac{y^\star}{\left(\frac{p^H}{rer}\right)^{\eta\star}}$$
$$x^H = y^H - x^{H\star}$$
$$y^C = \frac{x^H}{\omega(p^H)^{-\eta}}$$
$$x^F = (1-\omega)(p^F)^{-\eta}y^C$$
$$y^F = x^F$$
$$m = y^F\Xi^F$$

From (129)

$$tb = p^H x^{H\star} + p^{Co\star}y^{Co}rer - m\xi^m rer$$

From (93):

$$\upsilon = \frac{1}{a\pi}\begin{pmatrix} \gamma_D PD^D R^D d^F + +\gamma_{BB}PD^H R^{BB}q^{BB}bb^{Tot} + \mu_e G_e\left(\bar{\omega}^e\right)R^e q^K k \\ +\mu_I G_I\left(\bar{\omega}^I\right)R^H q^H h^I + \mu_H G_H\left(\bar{\omega}^H\right)\tilde{R}^H q^L l^H + \mu_F G_F\left(\bar{\omega}^F\right)\tilde{R}^F l^F \end{pmatrix}$$

Combine (91) and (128)

$$gdpn = y^C - \upsilon + tb$$

From their definitions:

$$s^g = \frac{g}{gdpn}$$
$$s^{Co} = \frac{y^{Co}p^{Co\star}rer}{gdpn}$$
$$s^{tb} = \frac{tb}{gdpn}$$

Supply of soverign debt instruments is inelastic, thus use ratios $\alpha_{BLG} = \frac{bl^G}{gdpnq^{BL}}$ and $\alpha_{SG} = \frac{bs^G}{gdpn}$

$$bl^G = \alpha_{BLG}\frac{gdpn}{q^{BL}}$$
$$bs^G = \alpha_{BSG}gdpn$$

From (102) and (103)

$$bl^{Pr} = -bl^G$$
$$bs^{Pr} = -bs^G$$
$$bs^U = \frac{bs^{Pr}}{\wp^U}$$
$$bb^U = \frac{bb^{tot}}{\wp^U}$$

Also, from (123), (124), (125), (126)

$$n^P = \frac{n}{2} = n^I = n^U = n^R$$

Next, we implement a numerical search for s^{b*} and $r^{h,k}$ (see Appendix D.1) using (78) and (128)

$$\xi^n = \frac{mc^W\lambda^W w}{\Theta\widetilde{n}^\varphi}$$
$$gdpn = c^P + c^I + i^K + i^H + g + tb$$

Then from its definition, we have

$$b^{*,Tot} = \frac{s^{b*}gdpn}{rer}$$

From (130)

$$ren^{*} = b^{*Tot}\left(1 - \frac{R^{\star}}{a\pi^{\star}}\right) - \frac{tb}{rer} + (1-\chi)\, p^{Co\star} y^{Co}$$

From $r^{h,k} = {}^{q^H h}/{}_{q^K k}$, (56) to (58):

$$\begin{aligned} h &= \frac{r^{h,k} q^K k}{q^H} \\ i^{AH} &= \frac{h a^{N_H}}{\xi^{ih}}\left[1 - \left(\frac{1-\delta_H}{a}\right)\right] \\ i^H &= i^{AH}\varphi_0^H \left[\frac{1 - \left(\frac{\rho^{\varphi H}}{a}\right)^{N_H+1}}{1 - \frac{\rho^{\varphi H}}{a}}\right] \end{aligned}$$

From (96)

$$h^P = h - h^I$$

From (101)

$$b^{*U} = \frac{b^{*Tot}}{\wp^U}$$

Then using the (exogenously given) ratio of long to short term instruments held by the unrestricted patient household, ω_{BL}

$$bl^u = \frac{\omega_{BL} * (bs^u + rer * b^{*U} + d^U) - bb^U q^{BB}}{q_{BL}}$$

We can then, using (98) results in long term bonds held by the restricted household of

$$bl^R = \frac{bl^{Pr} - \wp_U bl^U}{1 - \wp_U}$$

From (102)

$$bl^{CB} = 1$$

From (10) and (11) and the restricted household budget constraint (13)

$$aux_1 = (a)^{\sigma\eta_{\hat{C}}-1}(\xi^h)^{1-\eta_{\hat{C}}}\left(\frac{q^H}{\beta_P} - (1-\delta_H)\, a^{-\sigma} q^H\right)^{\eta_{\hat{C}}} \frac{(1-o_{\hat{C}})\left(1 - \frac{\phi_{hh}}{a}\right)}{o_{\hat{C}}\left(1 - \frac{\phi_c}{a}\right)}$$

$$\begin{aligned} h^R &= \frac{q^{BL} bl^R \left(\frac{R^{BL}}{a} - 1\right) + \frac{wn}{2}}{q^H - \frac{q^H}{a}(1-\delta_H) + aux_1} \\ c^R &= h^R aux_1 \end{aligned}$$

From (9):

$$\hat{c}^R = \left[(1-o_{\hat{C}})^{\frac{1}{\eta_{\hat{C}}}}\left(c^R\left(1 - \frac{\phi_c}{a}\right)\right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}} + (o_{\hat{C}})^{\frac{1}{\eta_{\hat{C}}}}\left(\xi^h \frac{h^R}{a}\left(1 - \frac{\phi_{hh}}{a}\right)\right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}}\right]^{\frac{\eta_{\hat{C}}}{\eta_{\hat{C}}-1}}$$

From (10):

$$\lambda^R = \left\{\left(\hat{c}^R\right)^{-\sigma}\right\}\left(\frac{(1-o_{\hat{C}})\,\hat{c}^R}{c^R\left(1 - \frac{\phi_c}{a}\right)}\right)^{\frac{1}{\eta_{\hat{C}}}}$$

From (97)

$$h^U = \frac{h^P - (1-\wp_U)h^R}{\wp_U}$$

From (2) and (3)

$$c^U = h^U(a)^{\sigma\eta_{\hat{C}}-1}(\xi^h)^{1-\eta_{\hat{C}}}\left(\frac{q^H}{\beta_P} - (1-\delta_H)\, a^{-\sigma} q^H\right)^{\eta_{\hat{C}}} \frac{(1-o_{\hat{C}})\left(1 - \frac{\phi_{hh}}{a}\right)}{o_{\hat{C}}\left(1 - \frac{\phi_c}{a}\right)}$$

From (1)

$$\hat{c}^U = \left[\left(1 - o_{\hat{C}}\right)^{\frac{1}{\eta_{\hat{C}}}} \left(c^U \left(1 - \frac{\phi_c}{a} \right) \right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}} + \left(o_{\hat{C}}\right)^{\frac{1}{\eta_{\hat{C}}}} \left(\xi^h \frac{h^U}{a} \left(1 - \frac{\phi_{hh}}{a} \right) \right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}} \right]^{\frac{\eta_{\hat{C}}}{\eta_{\hat{C}}-1}}$$

From (2)

$$\lambda^U = \left(\hat{c}^U\right)^{-\sigma} \left(\frac{\left(1 - o_{\hat{C}}\right) \hat{c}^U}{c^U \left(1 - \frac{\phi_c}{a}\right)} \right)^{\frac{1}{\eta_{\hat{C}}}}$$

From (76):

$$\lambda^P = \wp_U \lambda^U + (1 - \wp_U) \lambda^R$$

From (92):

$$c^P = \wp_U c^U + (1 - \wp_U) c^R; \qquad c = c^p + c^i$$

From (79) and (80):

$$\tilde{\chi}^U = \left(\hat{c}^U\right)^{\sigma}$$
$$\Theta^U = \tilde{\chi}^U \left(\hat{c}^U\right)^{-\sigma}$$
$$\tilde{\chi}^I = \left(\hat{c}^I\right)^{\sigma}$$
$$\Theta^I = \tilde{\chi}^I \left(\hat{c}^I\right)^{-\sigma}$$
$$\tilde{\chi}^R = \left(\hat{c}^R\right)^{\sigma}$$
$$\Theta^R = \tilde{\chi}^R \left(\hat{c}^R\right)^{-\sigma}$$
$$\Theta = \frac{\left(\omega_{UP}\Theta^U + (1 - \omega_U)\Theta^R\right) + \Theta^I}{2} = 1$$

From (75)

$$\lambda^W = \frac{\lambda^P + \lambda^I}{2}$$

From (7) and (105)

$$\epsilon^{L,S} = \beta_U R^{BL} a^{-\sigma} - 1$$

From (105) :

$$\zeta^L = \epsilon^{L,S}$$

From (85):

$$\tau = g + dia - bs^G \left(\frac{R}{a\pi} - 1 \right) - q^{BL} bl^G \left(\frac{R^{BL}}{a} - 1 \right) - \chi rerp^{Co\star} y^{Co}$$

From (86):

$$\alpha^T = \frac{\tau}{gdpn}$$

Finally, from (63), (72) and (82):

$$f^H = \frac{(\tilde{p}^H)^{-\epsilon_H} y^H mc^H}{1 - \beta_{UP}\theta_H a^{1-\sigma}}, \quad f^F = \frac{(\tilde{p}^F)^{-\epsilon_F} y^F mc^F}{1 - \beta_{UP}\theta_F a^{1-\sigma}}, \quad f^W = \frac{\tilde{w}^{-\epsilon_W(1+\varphi)} mc^W \tilde{n}}{1 - \left(\frac{(\omega_{UP}\beta^{UP} + (1-\omega_{UP})\beta^{RP}) + \beta_I}{2} \right) \theta_W a^{1-\sigma}}$$

D.1 Numerical solution for $(s^{b*}, r^{h,k})$

Iterate on $(s^{b*}, r^{h,k})$ until $\Delta \approx 0$

$$\Delta = \begin{bmatrix} \xi^n - \frac{mc^W \lambda^W w}{\Theta \tilde{n}^{\varphi}} \\ -gdpn + c^P + c^I + i^K + i^H + g + tb \end{bmatrix}$$

For each guess of $(s^{b*}, r^{h,k})$ we have

$$b^{*,Tot} = \frac{s^{b*} gdpn}{rer}$$

From $r^{h,k} = {}^{q^H h}/_{q^K k}$, (56) to (58):

$$h = \frac{r^{h,k} q^K k}{q^H}$$
$$i^{AH} = \frac{h a^{N_H}}{\xi^{ih}} \left[1 - \left(\frac{1-\delta_H}{a} \right) \right]$$
$$i^H = i^{AH} \varphi_0^H \left[\frac{1 - \left(\frac{\rho^{\varphi H}}{a} \right)^{N_H+1}}{1 - \frac{\rho^{\varphi H}}{a}} \right]$$

From (96)

$$h^P = h - h^I$$

From (101)

$$b^{*U} = \frac{b^{*Tot}}{\wp^U}$$

Then using the (exogenously given) ratio of long to short term instruments held by the unrestricted patient household, ω_{BL}

$$bl^u = \frac{\omega_{BL} * (bs^u + rer * b^{*U} + d^U) - bb^U q^{BB}}{q_{BL}}$$

We can then, using (98) results in long term bonds held by the restricted household of

$$bl^R = \frac{bl^{Pr} - \wp_U bl^U}{1 - \wp_U}$$

From (102)

$$bl^{CB} = 1$$

From (10) and (11) and the restricted household budget constraint (13)

$$aux_1 = (a)^{\sigma \eta_{\hat{C}} - 1} (\xi^h)^{1-\eta_{\hat{C}}} \left(\frac{q^H}{\beta_P} - (1-\delta_H)\, a^{-\sigma} q^H \right)^{\eta_{\hat{C}}} \frac{(1-o_{\hat{C}}) \left(1 - \frac{\phi_{hh}}{a}\right)}{o_{\hat{C}} \left(1 - \frac{\phi_c}{a}\right)}$$

$$h^R = \frac{q^{BL} bl^R \left(\frac{R^{BL}}{a} - 1 \right) + \frac{wn}{2}}{q^H - \frac{q^H}{a}(1-\delta_H) + aux_1}$$
$$c^R = h^R aux_1$$

From (9):

$$\hat{c}^R = \left[(1-o_{\hat{C}})^{\frac{1}{\eta_{\hat{C}}}} \left(c^R \left(1 - \frac{\phi_c}{a} \right) \right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}} + (o_{\hat{C}})^{\frac{1}{\eta_{\hat{C}}}} \left(\xi^h \frac{h^R}{a} \left(1 - \frac{\phi_{hh}}{a} \right) \right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}} \right]^{\frac{\eta_{\hat{C}}}{\eta_{\hat{C}}-1}}$$

From (10):

$$\lambda^R = \left\{ \left(\hat{c}^R \right)^{-\sigma} \right\} \left(\frac{(1-o_{\hat{C}})\, \hat{c}^R}{c^R \left(1 - \frac{\phi_c}{a}\right)} \right)^{\frac{1}{\eta_{\hat{C}}}}$$

From (97)

$$h^U = \frac{h^P - (1-\wp_U) h^R}{\wp_U}$$

From (2) and (3)

$$c^U = h^U (a)^{\sigma \eta_{\hat{C}} - 1} (\xi^h)^{1-\eta_{\hat{C}}} \left(\frac{q^H}{\beta_P} - (1-\delta_H)\, a^{-\sigma} q^H \right)^{\eta_{\hat{C}}} \frac{(1-o_{\hat{C}}) \left(1 - \frac{\phi_{hh}}{a}\right)}{o_{\hat{C}} \left(1 - \frac{\phi_c}{a}\right)}$$

From (1)

$$\hat{c}^U = \left[(1-o_{\hat{C}})^{\frac{1}{\eta_{\hat{C}}}} \left(c^U \left(1 - \frac{\phi_c}{a} \right) \right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}} + (o_{\hat{C}})^{\frac{1}{\eta_{\hat{C}}}} \left(\xi^h \frac{h^U}{a} \left(1 - \frac{\phi_{hh}}{a} \right) \right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}} \right]^{\frac{\eta_{\hat{C}}}{\eta_{\hat{C}}-1}}$$

From (2)

$$\lambda^U = \left(\hat{c}^U\right)^{-\sigma} \left(\frac{\left(1 - o_{\hat{C}}\right) \hat{c}^U}{c^U \left(1 - \frac{\phi_c}{a}\right)} \right)^{\frac{1}{\eta_{\hat{C}}}}$$

From (76):

$$\lambda^P = \wp_U \lambda^U + (1 - \wp_U) \lambda^R$$

From (92):

$$c^P = \wp_U c^U + (1 - \wp_U) c^R; \qquad c = c^p + c^i$$

From (79) and (80):

$$\tilde{\chi}^U = \left(\hat{c}^U\right)^{\sigma}$$
$$\Theta^U = \tilde{\chi}^U \left(\hat{c}^U\right)^{-\sigma}$$
$$\tilde{\chi}^I = \left(\hat{c}^I\right)^{\sigma}$$
$$\Theta^I = \tilde{\chi}^I \left(\hat{c}^I\right)^{-\sigma}$$
$$\tilde{\chi}^R = \left(\hat{c}^R\right)^{\sigma}$$
$$\Theta^R = \tilde{\chi}^R \left(\hat{c}^R\right)^{-\sigma}$$
$$\Theta = \frac{\left(\omega_{UP} \Theta^U + (1 - \omega_U) \Theta^R\right) + \Theta^I}{2} = 1$$

From (75)

$$\lambda^W = \frac{\lambda^P + \lambda^I}{2}$$

Check if $\Delta = 0$

$$\Delta = \begin{bmatrix} \xi^n - \frac{mc^W \lambda^W w}{\Theta \tilde{n}^{\varphi}} \\ -gdpn + c^P + c^I + i^K + i^H + g + tb \end{bmatrix}$$